Math 1220  
Fall 2002  
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**EXAM III**  
Wednesday, November 27, 2002

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(5 pts) 1. Write the following polar equation in cartesian form $r = 4\cos \theta + 3\sin \theta$. What geometric object does this equation determine?

**Solution**

If we multiply both sides of the equation by $r$ we obtain the equation $r^2 = 4r\cos \theta + 3r\sin \theta$. This equation written in cartesian form is then $x^2 + y^2 = 4x + 3y$, since $x = r\cos \theta$ and $y = r\sin \theta$. The geometric object that this equation represents is a circle.

(7 pts) 2. Find the slope of the tangent to the graph of $r = 1 + \sin \theta$ at the point of the graph where $\theta = \frac{\pi}{2}$.

**Solution**

Since $y = r\sin \theta$ this then implies that $y = \sin \theta + \sin^2 \theta$. Hence $\frac{dy}{d\theta} = \cos \theta + 2\sin \theta \cos \theta$. This implies that $\frac{dy}{d\theta}$ at $\theta = \frac{\pi}{2}$ is zero. Hence the slope of the tangent line to the graph is zero.

(6 pts) 3. Solve the differential equation $y'' + 4y' + 4y = 3e^x$.

**Solution**

The first thing to do is to find the general solution to the homogeneous differential equation $y'' + 4y' + 4y = 0$. The auxiliary equation for this is $r^2 + 4r + 4 = 0$. The solution to this equation is $r = 2$. Hence the general solution is $C_1e^{-2x} + C_2xe^{-2x}$. Now find the particular solution. Since the function on the right hand side is $e^x$ and $e^x$ is not a solution to the homogeneous equation, then the particular solution has the form $Ae^x$. If you now substitute the particular solution into the equation, you will find that $A = 1/3$. Hence the solution to the differential equation is $C_1e^{-2x} + C_2xe^{-2x} + \frac{1}{3}e^x$.

(8 pts) 4. Find the Taylor polynomial of order 3 based at $\pi$ for the function $\sin x$.

**Solution**

The Taylor polynomial of order three is given by the polynomial

$$f(x) + f'(\pi)(x - \pi) + \frac{f''(\pi)}{2}(x - \pi)^2 + \frac{f^3(\pi)}{6}(x - \pi)^3,$$

where $f(x) = \sin x$. Now $f'(x) = \cos x$, $f''(x) = -\sin x$ and $f^3(x) = -\cos x$. Hence the Taylor polynomial of order three is...
(10 pts) 5. Find the area of the region enclosed by the graph \( r = 3 + \cos \theta \)

**Solution**

The area of the region is given by the integral \( \int_0^\pi (3 + \cos \theta)^2 \, d\theta \). Expanding this out and using the linearity of the integral we deduce that the area is equal to \( \int_0^\pi 9 \, d\theta + \int_0^\pi \cos \theta \, d\theta + \int_0^\pi \cos^2 \theta \, d\theta \). The next step is to replace \( \cos^2 \theta \) by \( \frac{\cos 2\theta + 1}{2} \). Then if you do the integration you should obtain the answer \( \frac{19\pi}{2} \).

(7 pts) 6. Solve the differential equation \( y'' - 2y' + 5y = 0 \).

**Solution**

The auxiliary equation for this differential equation is \( r^2 - 2r + 5 \). The solutions to this equation are \( 1 \pm 2i \). Hence the solution to the equation is \( y = C_1 e^x \cos 2x + C_2 e^x \sin 2x \).