MATHEMATICS 2210-3. Homework 2.

January 17, 2001

1. Problems # 2 (c,e), 5 (a), 18 from Section 13.3.

   Problem # 2 (Each item is worth 5 points). Let \( \overrightarrow{a} = (3, -1) \), \( \overrightarrow{b} = (1, -1) \), \( \overrightarrow{c} = (0, 5) \). Find each fo the following:

   (c). \( (\overrightarrow{a} + \overrightarrow{b}) \cdot \overrightarrow{c} \).

   Solution. \( \overrightarrow{v} = \overrightarrow{a} + \overrightarrow{b} = (3, -1) + (1, -1) = (4, -2) \). \( \overrightarrow{v} \cdot \overrightarrow{c} = (4, -2) \cdot (0, 5) = -10 \). Hence \( (\overrightarrow{a} + \overrightarrow{b}) \cdot \overrightarrow{c} = -10 \).

   (e). \( |\overrightarrow{b}| \overrightarrow{b} \cdot \overrightarrow{a} \).

   Solution. \( \overrightarrow{b} \cdot \overrightarrow{a} = (1, -1) \cdot (3, -1) = 3 + 1 = 4 \). \( |\overrightarrow{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2} \). Hence \( |\overrightarrow{b}| \overrightarrow{b} \cdot \overrightarrow{a} = 4\sqrt{2} = \sqrt{32} \).

   Problem # 5 (a). (5 points) Write the vector \( \overrightarrow{AB} \) in the form \( a_1 \overrightarrow{i} + a_2 \overrightarrow{j} \). Here \( A \) has coordinates \((2, 2)\), and \( B \) has coordinates \((-3, 4)\).

   Solution. \( \overrightarrow{AB} = (-3, 4) - (2, 2) = (-5, 2) = -5 \overrightarrow{i} + 2 \overrightarrow{j} \).

   Problem # 18. (5 points) For what numbers \( c \) are \( 2c \overrightarrow{i} - 8 \overrightarrow{j} \) and \( 3 \overrightarrow{i} + c \overrightarrow{j} \) orthogonal?

   Solution. \( (2c \overrightarrow{i} - 8 \overrightarrow{j}) \cdot (3 \overrightarrow{i} + c \overrightarrow{j}) = 6c - 8c = -2c \). The vectors are orthogonal if and only if their dot product is zero. Hence \(-2c = 0\), and thus \( c = 0 \).

2. Problems # 24, 32 from Section 13.4. Each problem is worth 10 points.

   Problem # 24. The position of a moving particle at time \( t \) is given by \( \overrightarrow{r}(t) \). Find the velocity and acceleration vectors \( \overrightarrow{v} \) and \( \overrightarrow{a} \) and speed at the time \( t_1 = 1/2 \). Sketch a portion of the graph of \( \overrightarrow{r}(t) \) containing \( P \) with \( \overrightarrow{OP} = r(t_1) \) and sketch \( v(t_1), a(t_1) \) with their initial points at \( P \).

   Here \( r(t) = (3t^2 - 1) \overrightarrow{i} + t \overrightarrow{j} \).
Solution. $\vec{v}(t) = \frac{d}{dt} \vec{r}(t) = 6t \vec{i} + \vec{j}$, $a(t) = \frac{d}{dt} \vec{v}(t) = 6 \vec{i}$. At $t_1 = 1/2$ we get:

$$\vec{v}(0.5) = 3 \vec{i} + \vec{j}, a(0.5) = 6 \vec{i}.$$  

Speed equals $|\vec{v}(0.5)| = \sqrt{9 + 1} = \sqrt{10}$. See Figure 1.

![Figure 1: Problem 24.](image)

Problem # 32. Find the velocity vector $\vec{v}(t)$ and the position vector $\vec{r}(t)$ given that

$$\vec{a}(t) = t \vec{j}, \vec{v}(0) = \vec{i} + 2 \vec{j}, \vec{r}(0) = \vec{0}. $$

Solution. $\vec{v}(t) = \int_0^t a(t) \, dt = \int_0^t t \vec{j} \, dt = c \vec{i} + d \vec{j} + t^2/2 \vec{j}$. Here $c,d$ are scalars to be computed. Note that we have two unknown scalars since our vectors have two components. Since $\vec{v}(0) = \vec{i} + 2 \vec{j}$, we get: $c = 1, d = 2$. Hence $\vec{v}(t) = \vec{i} + (2 + t^2/2) \vec{j}$.

Similarly,

$$\vec{r}(t) = \int_0^t \vec{v}(t) \, dt = t \vec{i} + (2t + t^3/6) \vec{j} + a \vec{i} + b \vec{j}.$$  

To find $a, b$ we use that $\vec{r}'(0) = \vec{0}$ and hence $a = 0, b = 0$. Thus

$$\vec{r}(t) = t \vec{i} + (2t + t^3/6) \vec{j}. $$