1. Problems # 14, 22 from Section 16.2.

**Problem 14.** (10 points) Evaluate the integral \( \iint_R (x^2 + y^2) \, dx \, dy \), where \( R \) is the rectangle \(-1 \leq x \leq 1, -1 \leq y \leq 1\).

**Solution.**

\[
\begin{align*}
\iint_R (x^2 + y^2) \, dx \, dy &= \int_{-1}^{1} \int_{-1}^{1} (x^2 + y^2) \, dx \, dy \\
&= \int_{-1}^{1} \left[ \frac{x^3}{3} \right]_{-1}^{1} + \left[ y^2 x \right]_{-1}^{1} \, dy \\
&= \int_{-1}^{1} \left( \frac{2}{3} y + \frac{2}{3} y^3 \right) \, dy = \frac{8}{3}.
\end{align*}
\]

**Problem 22.** (10 points) Find the volume of the given solid. Sketch the solid.

The solid is under the plane \( z = 2x + 3y \) over the rectangle \( R: 1 \leq x \leq 2, 0 \leq y \leq 4 \).

**Solution.** Below is the sketch, see Figure 1.

The volume equals

\[
\begin{align*}
\iiint_R (2x + 3y) \, dx \, dy &= \int_{1}^{2} \int_{0}^{4} (2x + 3y) \, dy \, dx \\
&= \int_{1}^{2} \left[ 2xy + \frac{3}{2} y^2 \right]_{0}^{4} \, dx \\
&= \int_{1}^{2} [8x + 24] \, dx = [4x^2 + 24x]_{1}^{2} = 16 + 48 - 4 - 24 = 36.
\end{align*}
\]

2. Problems # 10, 16, 22 from Section 16.3.

**Problem 10.** (10 points) Evaluate the integral

\[
\int_{0}^{\pi/2} \int_{0}^{\sin(y)} e^x \cos(y) \, dx \, dy
\]

**Solution.** The integral equals

\[
\begin{align*}
\int_{0}^{\pi/2} \int_{0}^{\sin(y)} e^x \cos(y) \, dx \, dy &= \int_{0}^{\pi/2} \left[ e^{\sin(y)} - 1 \right] \cos(y) \, dy \\
&= \left[ e^{\sin(y)} \right]_{0}^{\pi/2} - \left[ \sin(y) \right]_{0}^{\pi/2} = e - 1 + 1 = e.
\end{align*}
\]

**Problem 16.** (10 points) Evaluate the integral

\[
\iint_S (x^2 - xy) \, dx \, dy
\]
where \( S \) is the (bounded) region \( S \) between the curves \( y = x \) and \( y = 3x - x^2 \).

**Solution.** For points in \( S \) the \( x \)-coordinate varies between 0 and 2, the curve \( y = 3x - x^2 \) is above the line \( y = x \).

Thus the integral equals

\[
\int_0^2 \int_x^{3-x^2} (x^2 - xy) dy \, dx = \int_0^2 \int_x^{3-x^2} \left[ x^2 (3x - x^2) - x(3x - x^2)^2/2 \right] \, dx = \int_0^2 \int_x^{3-x^2} \left[ x^3 - x^3/2 \right] \, dx = \int_0^2 [3x^3 - x^4 - x(9x^2 - 6x^3 + x^4)/2 - x^3/2] \, dx = \int_0^2 [-2x^3 + 2x^4 - x^5/2] \, dx = \left[ -x^4/2 + \frac{2}{5}x^5 - x^6/12 \right]_0^2 = \frac{232}{15}.
\]

**Problem 22.** (20 points) Sketch the given solid. Compute its volume via integration. The solid \( V \) is in the first octant, bounded by the coordinate planes and the planes \( 2x + y - 4 = 0 \) and \( 8x + y - 4z = 0 \).

**Solution.** We first sketch the planes \( 2x + y - 4 = 0 \) and \( 8x + y - 4z = 0 \). In the first equation the coordinate \( z \) is absent, hence it is a vertical plane, it intersects the \( xy \) plane along the line \( 2x + y - 4 = 0 \) and it intersects the \( xz \)-plane along the line \( x = 2 \), it intersects the \( yz \)-plane along the line \( y = 4 \).
To sketch the other plane note that it intersects the first octant only at the origin, thus we sketch only its intersections with the \(xz\)-plane and the \(yz\) plane. These are the lines \(8x - 4z = 0\) (or \(z = 2x\)) and \(y - 4z = 0\).

Thus the solid \(V\) is the region below the plane \(8x + y - 4z = 0\) and above the triangle \(S\) with the vertices \((2,0),(0,4),(0,0)\) in the \(xy\)-plane.

The plane \(8x + y - 4z = 0\) is the graph of the function \(z = f(x,y) = 2x + y/4\). Thus the volume equals

\[
\iint_{S} [2x + y/4] dy dx = \int_{0}^{2} \int_{0}^{1-2x} [2x + y/4] dy dx = \\
\int_{0}^{2} [2xy + y^2/8]_{0}^{1-2x} dx = \int_{0}^{2} [2x(4 - 2x) + (4 - 2x)^2/8] dx = \\
\int_{0}^{2} [8x - 4x^2 + (2 - x)^2/2] dx = [4x^2 - 4x^3/3 - (2 - x)^3/6]_{0}^{2} = \\
(16 - 32/3) + 4/3 = 16 - 28/3 = 20/3.
\]

3. Problems # 2, 16 from Section 16.4.

**Problem 2.** (10 points) Compute the integral

\[
\int_{0}^{\pi/2} \int_{0}^{\sin(\theta)} r d\theta dr.
\]

**Solution.** The integral equals

\[
\frac{1}{2} \int_{0}^{\pi/2} [\sin^2(\theta)]_{0}^{\pi/2} d\theta = \frac{1}{2} \int_{0}^{\pi/2} \sin^2(\theta) d\theta = \frac{1}{2} \left[ \frac{\theta}{2} - \frac{1}{4} \sin(2\theta) \right]_{0}^{\pi/2} = \frac{\pi}{8}.
\]
**Problem 16.** (10 points) Compute the integral

\[ \int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2) \, dx \, dy. \]

**Solution.** The region of integration is the quarter of the unit disk (with center at the origin) located in the first coordinate quadrant. Hence in the polar coordinates this region is given by the inequalities \( 0 \leq r \leq 1, \quad 0 \leq \theta \leq \pi/2 \). See Figure 4. By changing to the polar coordinates we get:

\[ \int_0^1 \int_0^{\pi/2} \sin(r^2) \, r \, d\theta \, dr = \]

\[ \frac{\pi}{2} \int_0^1 \sin(r^2) \, r \, dr = \frac{\pi}{4} \left[ \cos(r^2) \right]_0^1 = \frac{\pi}{4} (\cos(1) - 1) = \frac{\pi}{4} (1 - \cos(1)). \]
Figure 4: