MATHEMATICS 2210-3. Homework 10: Solution.

March 19, 2001

1. Problems # 2, 6, 16 from Section 16.1.

Problem 2. (5 points) Let $R$ be the rectangle $1 \leq x \leq 4, 0 \leq y \leq 2$. Compute the integral $\iint_R f(x, y)dx\,dy$ for the function

$$f(x, y) = \begin{cases} 
2 & \text{if } 1 \leq x < 3, 0 \leq y \leq 2 \\
3 & \text{if } 3 \leq x \leq 4, 0 \leq y \leq 2 
\end{cases}$$

Solution. The function $f$ equals 2 on the rectangle $R_1$ which has the dimensions $2 \times 2$; the function $f$ equals 3 on the rectangle $R_2$ which has the dimensions $1 \times 2$. Hence the integral $\iint_R f(x, y)dx\,dy$ equals $2 \cdot 2 \cdot 2 + 3 \cdot 1 \cdot 2 = 8 + 6 = 14$.

Problem 6. (5 points) Assuming $\iint_R f(x, y)dx\,dy = 3$, $\iint_R g(x, y)dx\,dy = 5$, compute

$$\iint_R 2f(x, y) + 5g(x, y)dx\,dy.$$ 

Solution.

$$\iint_R 2f(x, y) + 5g(x, y)dx\,dy = 2 \iint_R f(x, y)dx\,dy + 5 \iint_R g(x, y)dx\,dy = 2 \cdot 3 + 5 \cdot 5 = 31.$$ 

Problem 16. (10 points) Using elementary geometry compute the integral $\iint_R (1 + x)dx\,dy$ where $R$ is the rectangle $0 \leq x \leq 2, 0 \leq y \leq 1$.

Solution. The integral is the volume of the solid between the rectangle $R$ and the plane $z = x + 1$. This solid is the prism with the height 1 whose base is the trapezoid (contained in the $xz$-plane). This trapezoid has the height 2 (parallel to the $x$-axis) and the bases of the lengths 1 and 3. Thus the area of the trapezoid equals $2(1 + 3)/2 = 4$. The volume of the prism is $1 \cdot 4 = 4$ (height times the area of the base). Thus the integral equals 4.