**Definition**

- **Gradient**
  
  \[ \nabla \]

- **Divergence**
  
  \[ \nabla \cdot \]

- **Curl**
  
  \[ \nabla \times \]

- **5 species of second derivatives**

**Theorem**

- **Curl-less or irrotational fields**
  
  \[ \nabla \cdot \]

- **Divergence-less or solenoidal fields**

- **Gradient theorem**
  
  \[ \nabla \times \]

- **Green's theorem**
The gradient $\nabla T$ points in the direction of maximum increase of the function $T$.

$$\nabla T \equiv \hat{x} \frac{\partial T}{\partial x} + \hat{y} \frac{\partial T}{\partial y} + \hat{z} \frac{\partial T}{\partial z}$$

The magnitude $|\nabla T|$ is the slope along this direction.

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$$\nabla \cdot v = (\hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}) \cdot (v_x \hat{x} + v_y \hat{y} + v_z \hat{z})$$

$$\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$$

The divergence is a measure of how much the vector function $v$ spreads out from the point in question.

By applying $\nabla$ twice we can construct five species of second derivatives.

1. divergence of a gradient $\nabla \cdot (\nabla T) = \nabla^2$ (Laplacian)
2. curl of a gradient $\nabla \times (\nabla T) = 0$ (always)
3. gradient of a divergence $\nabla (\nabla \cdot v)$ (seldom occurs)
4. divergence of a curl $\nabla \cdot (\nabla \times v) = 0$ (always)
5. curl of a curl $\nabla \times (\nabla \times v) = \nabla (\nabla \cdot v) - \nabla^2 v$

The curl is a measure of how much the vector field “curls around” the point in question.

For a given vector field $F$ the following statements are equivalent, i.e. each implies the others.

1. $\nabla \cdot F = 0$ everywhere
2. $\int F \cdot da$ is independent of surface
3. $\oint F \cdot da = 0$ over any closed surface
4. $F = \nabla \times A$ for some vector potential $A$

For a given vector field $F$ the following statements are equivalent, i.e. each implies the others.

1. $\nabla \times F = 0$ everywhere
2. $\int_a^b F \cdot dl$ is path independent
3. $\oint_a^b F \cdot dl = 0$ on any closed loop
4. $F = -\nabla V$ for some scalar potential $V$

$$\int (\nabla \cdot A) dV = \oint A \cdot da$$

$$\int_a^b (\nabla f) \cdot dl = f(b) - f(a)$$
Theorem

Stokes’ theorem

Electrodynamics
\[ \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l} \]