1. [10 pts.] If possible, compute $A^{-1}$ for

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$
2. Solve the following homogeneous, linear DEs with constant coefficients:
   
   (a) [5 pts.] \( y'' - 5y' + 6y = 0 \)

   (b) [5 pts.] \( y''' - 5y'' + 7y' - 3y = 0 \)

   (c) [5 pts.] \( y'' + 4y' + 40y = 0 \)
3. [8 pts.] Is the following set of vectors from \( \mathbb{R}^4 \) linearly dependent or linearly independent? Circle one answer and justify your answer.

\[
S = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}
\]

Linearly Dependent  
Linearly Independent

4. [5 points (bonus)] Is the set \( S \) a basis for the vector space \( \mathbb{R}^4 \)? Circle one answer and justify your answer. (You won’t receive any credit without justification.)

YES  
NO
5. [15 pts.] Use the method of undetermined coefficients to find a general solution to

\[ y'' + 4y = 8\cos 2x. \]  \hspace{1cm} (1)
6. [9 pts.] Recall that $M_{2 \times 2}$, the set of $2 \times 2$ matrices, where

$$M_{2 \times 2} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a, b, c, d \in \mathbb{R} \right\}$$

forms a vector space. Show that $U$, the subset of upper triangular $2 \times 2$ matrices, where

$$U = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| a, b, c \in \mathbb{R} \right\} \subset M_{2 \times 2},$$

is a subspace of $M_{2 \times 2}$.

7. [1 pts.] What is the dimension of $U$?
8. Circle or box the letter corresponding to the correct trial particular solution, $y_p$, for each given equation.

(a) [3 pts.] $y'' - 6y' - 7y = x$

A. $y_p = Ax$
B. $y_p = A + Bx + Cx^2$
C. $y_p = (A + Bx)e^x$
D. $y_p = A + Bx$

(b) [3 pts.] $y'' - 4y' + 4y = e^{2x}$

A. $y_p = Ae^{2x}$
B. $y_p = Axe^{2x}$
C. $y_p = Ax^2 e^{2x}$
D. $y_p = Ax^3 e^{2x}$
(c) [3 pts.] \( y''' - y' = x \)

A. \( y_p = Bx + Cx^2 \)  
B. \( y_p = A + Bx \)  
C. \( y_p = Bx \)  
D. \( y_p = A \)

(d) [3 pts.] \( (D^2 + 9)^2 y = 5 \cos 3x \)

A. \( y_p = A \cos 3x + B \sin 3x \)  
B. \( y_p = (A + Bx) \cos 3x + (C + Dx) \sin 3x \)  
C. \( y_p = Bx \cos 3x + Dx \sin 3x \)  
D. \( y_p = Cx^2 \cos 3x + Fx^2 \sin 3x \)
9. [20 pts.] True or False. Circle one.

(a) T F The following algebraic system has infinitely many solutions.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(b) T F The following algebraic system has infinitely many solutions.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

(c) T F If the determinant of a matrix is 0 then that matrix is invertible.

(d) T F A fourth order, linear, homogeneous DE with constant coefficients has exactly four linearly independent solutions.

(e) T F The solution space of a linear differential equation is a vector subspace of \( \mathcal{F} \), the vector space of all real valued functions.

(f) T F The driven mass—spring—dashpot system corresponding to the following DE is underdamped.

\[x'' + 5x' + 25x = \cos 2t\]

(g) T F If \( A \) and \( B \) are \( n \times n \) matrices, then \( AB = BA \).

(h) T F The following two linear operators are equivalent, that is \( L_1 = L_2 \).

\[L_1 = (D - 3)(D + 2) \quad L_2 = (D + 2)(D - 3)\]

(i) T F Suppose \( A \) is an \( n \times n \) matrix. If \( A \) is invertible, then for every \( n \)-vector \( \vec{b} \), the system \( A\vec{x} = \vec{b} \) has a unique solution.

(j) T F The following system will exhibit resonance.

\[x'' + 9x = \sin 3t\]
10. The are two ways you can fail to earn the points for the last two questions:

1. not providing an answer,
2. not writing an integer between 1 and 10 inclusive.

(a) [5 pts.] Rate the difficulty of this exam on a scale of 1 to 10, where 1 is very easy and 10 is very difficult.

(a) ____________

(b) [5 pts.] Rate the fairness of this exam on a scale of 1 to 10, where 1 is completely unfair and 10 is completely fair.

(b) ____________
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