1. [10 pts.] Find the general solution to

\[ y'' + 2y' + 10y = 0. \]

2. [10 pts.] Determine the linear, homogeneous differential equation with constant coefficients which has solutions:

\[ y_1 = e^{-x} \quad y_2 = e^x \quad y_3 = xe^x. \]

*Hint:* What would the roots of the characteristic equation need to be in order to get these solutions?
3. [15 pts.] Use Gauss-Jordan elimination to solve the system

\[
\begin{align*}
    x_1 &+ x_2 + x_3 + x_4 = 12 \\
    x_2 &- x_3 + 4x_4 = 5 \\
    3x_1 &+ 2x_2 + 4x_3 - x_4 = 31
\end{align*}
\]
4. [5 pts.] The matrix \( A = \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \end{bmatrix} \) projects any vector in \( \mathbb{R}^3 \) to the \( xy \)-plane of \( \mathbb{R}^3 \), because
\[
\begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\
y \\
z \end{bmatrix} = \begin{bmatrix} x \\
y \\
0 \end{bmatrix}.
\]
In other words, \( A \) sets the \( z \) component of any vector to 0. What is the solution space of the following homogeneous equation?
\[
\begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\
y \\
z \end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \end{bmatrix} \tag{1}
\]
A. All vectors in the \( xy \)-plane
B. All vectors in the \( xz \)-plane
C. All vectors in the \( yz \)-plane
D. Only the zero vector
E. \( \mathbb{R}^3 \)
F. The \( z \)-axis

5. [5 pts.] Recall that equation (1) above is exactly equivalent to the equation
\[
x \begin{bmatrix} 1 \\
0 \\
0 \end{bmatrix} + y \begin{bmatrix} 0 \\
1 \\
0 \end{bmatrix} + z \begin{bmatrix} 0 \\
0 \\
0 \end{bmatrix} = \begin{bmatrix} 0 \\
0 \\
0 \end{bmatrix}.
\]
What is the span of the three column vectors of matrix \( A \)?
A. All vectors in the \( xy \)-plane
B. All vectors in the \( xz \)-plane
C. All vectors in the \( yz \)-plane
D. Only the zero vector
E. \( \mathbb{R}^3 \)
F. The \( z \)-axis
6. [15 pts.] Determine the correct differential equation for the free (undriven), mass–spring–dashpot system pictured and solve it for $x(t)$.

Assume: $m = 1$, $c = 2$, $k = 10$, $x(0) = 5$, $x'(0) = 0$. 

\[
mx''(t) + cx'(t) + kx(t) = 0
\]
7. [10 pts.] Circle or box the letter corresponding to the correct trial particular solution, \( y_p \), for each given equation.

(a) \( y'' - y' - 6y = 2 + x \)

A. \( y_p = Ax \)
B. \( y_p = A + Bx \)
C. \( y_p = Ae^{-2x} + Be^{3x} \)
D. \( y_p = (A + Bx)e^x \)

(b) \( y'' + 3y' + 2y = e^{-x} \)

A. \( y_p = Ae^{-x} \)
B. \( y_p = Axe^{-x} \)
C. \( y_p = Ae^{-x} + Be^{-2x} \)
D. \( y_p = x(Ae^{-x} + Be^{-2x}) \)

(c) \( y'' + 3y' + 2y = xe^{4x} \)

A. \( y_p = Ae^{4x} \)
B. \( y_p = Axe^{4x} \)
C. \( y_p = (A + Bx)e^{4x} \)
D. \( y_p = (A + Bx + Cx^2)e^{4x} \)

(d) \( y'' + 9y = 5 \cos 3x \)

A. \( y_p = A \cos 3x \)
B. \( y_p = (A + Bx) \cos 3x \)
C. \( y_p = A \cos 3x + B \sin 3x \)
D. \( y_p = x(A \cos 3x + B \sin 3x) \)
8. [20 pts.] True or False. Circle one.

(a) T F A system of linear, algebraic equations may have two and only two distinct solutions.

(b) T F A homogeneous system of linear algebraic equations always has at least one solution.

(c) T F The solution space of a homogeneous, linear differential equation is a vector subspace of $\mathcal{F}$, the vector space of all real valued functions.

(d) T F Multiplying a row of a matrix by any real number is a valid elementary row operation.

(e) T F $\left| \begin{array}{ccc} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{array} \right| = 6$

(f) T F The following equation of a mass—spring—dashpot system is critically damped.

$$x'' + 5x' + \frac{25}{4}x = 0$$

(g) T F Let the set $B$ be a basis for the vector space $V$. If you remove any vector from $B$ then it will no longer span $V$.

(h) T F Let the set $B$ be a basis for a vector space $V$. If you add any vector to $B$, then its vectors will be linearly dependent.

(i) T F The matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 9 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is in reduced row–echelon form.

(j) T F Suppose a square matrix, $A$ has determinant, $|A| = 5$. If we interchange the first two rows of $A$ to make $A'$, then $|A'| = -5$. 
9. The are two ways you can fail to earn the points for the last two questions:

1. not providing an answer,
2. not writing an integer between 1 and 10 inclusive.

(a) [5 pts.] Rate the difficulty of this exam on a scale of 1 to 10, where 1 is very easy and 10 is very difficult.

(a) __________

(b) [5 pts.] Rate the fairness of this exam on a scale of 1 to 10, where 1 is completely unfair and 10 is completely fair.

(b) __________
Scratch Paper

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