1. Fundamental Theorem For Line Integrals
Let \( C \) be a piecewise smooth curve given parametrically by \( \mathbf{r} = \mathbf{r}(t), \ a \leq t \leq b \), which begins at \( \mathbf{a} = b\mathbf{r}(a) \) and ends at \( \mathbf{b} = \mathbf{r}(b) \). If \( f \) is continuously differentiable on an open set containing \( C \), then
\[
\int_C \nabla f(\mathbf{r}) \cdot d\mathbf{r} = f(\mathbf{b}) - f(\mathbf{a})
\]

2. Green’s Theorem
Let \( C \) be a smooth, simple closed curve that forms the boundary of a region \( S \) in the \( xy \)-plane. If \( M(x, y) \) and \( N(x, y) \) are continuous and have continuous partial derivatives on \( S \) and its boundary \( C \), then
\[
\iint_S \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) = \oint_C Mdx + Ndy
\]

3. Gauss’s Theorem
Let \( \mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k} \) be a vector field such that \( M, N, P \) have continuous first-order partial derivatives on a solid \( S \) with boundary \( \partial S \). If \( \mathbf{n} \) denotes the outer unit normal to \( \partial S \), then
\[
\iint_{\partial S} \mathbf{F} \cdot \mathbf{n}dS = \iiint_S \text{div} \mathbf{F}dV
\]
In other words, the flux of \( \mathbf{F} \) across the boundary of a closed region in three space is the triple integral of its divergence over that region.

4. Stokes’s Theorem
Let \( S, \partial S, \) and \( \mathbf{n} \) be as indicated in the textbook, and suppose that \( \mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k} \) is a vector field, with \( M, N, P \) having continuous first-order partial derivatives on \( S \) and its boundary \( \partial S \). If \( \mathbf{T} \) denotes the unit tangent vector to \( \partial S \), then
\[
\oint_{\partial S} \mathbf{F} \cdot d\mathbf{T} = \iint_S (\text{curl} \mathbf{F}) \cdot \mathbf{n}dS
\]