MATH 2210-4 Spring 2010
Second Mock Exam
INSTRUCTOR: H.-PING HUANG

LAST NAME ____________________________
FIRST NAME __________________________
ID NO. ________________________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE SPECIFIED METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1  20  ______
PROBLEM 2  20  ______
PROBLEM 3  20  ______
PROBLEM 4  20  ______
PROBLEM 5  20  ______
TOTAL  100  ______
PROBLEM 1

(20 pt) Match the given equation with the verbal description of the surface:

- A. Plane
- B. Circular Cylinder
- C. Half Plane
- D. Sphere
- E. Cone
- F. Elliptic or Circular Paraboloid

1. $\phi = \pi/3$
2. $r = 4$
3. $\rho \cos(\phi) = 4$
4. $r = 2\cos(\theta)$
5. $\rho = 4$
6. $r^2 + z^2 = 16$
7. $\rho = 2\cos(\phi)$
8. $z = r^2$
9. $\theta = \pi/3$
PROBLEM 2

(20 pt) If \( \sin(2x + 3y + z) = 0 \) at the point \((0, 0, 0)\), use implicit differentiation to find the first partial derivatives \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \).

Find the equation of the tangent plane to the surface at the point \((0, 0, 0)\). Use it to estimate the value of \( z \) when \( x = 0.1 \) and \( y = 0.1 \).
PROBLEM 3

(20 pt) Find the limit (enter “DNE” if the limit does not exist).

\[
\lim_{(x,y) \to (0,0)} \frac{-2x^2 + 8y^2}{\sqrt{-2x^2 + 8y^2 + 1} - 1}
\]
PROBLEM 4

(20 pt) Suppose $z = f(x, y) = x/y + y/x$, $P = (2, 1)$.

A. Find the gradient of $f$ at the point $P$.

B. Compute the directional derivative of $f$ at the point $P$ in the direction of the origin.

C. What is the maximum rate of change of $f$ at the point $P$?

D. In which direction would $f$ increase most rapidly at the point $P$?

E. At what angle above the horizontal would $f$ change in that direction at the point $P$?
PROBLEM 5

(20 pt) Find the differential of the function.

\[ w = x^3 \sin yz^2 \]

If \( x = e^{2t} \), \( y = 2 + \sin 2t \), and \( z = f(x, y) = 3x^2 - 4y^2 \), what is \( \frac{dw}{dt} \) at \( t = 0? \)