Section 6.1: Simple Interest; Sequences

Definitions:

- The **simple interest** is given by
  \[ I = Prt, \]
  where
  
  \[ I = \text{interest (in dollars)} \]
  \[ P = \text{principal (in dollars)} \]
  \[ r = \text{interest rate in certain period (given as a decimal)} \]
  \[ t = \text{time} \]

  Note: The time measurements for \( r \) and \( t \) must agree. (e.g., **annual** interest rate for \( r \) and \( t \) in years)

- The **future value** of an investment or a loan \( S \) is given by
  \[ S = P + I = P + Prt = P(1 + rt), \]
  where \( P \) is the principal (in dollars) and \( I \) is the interest (in dollars).

- The principal of a loan is also called the **face value** or the **present value** of the loan.

Ex.1 (#4): $1800 is invested for 9 months at an annual simple interest rate of 15%.

(a) How much interest will be earned?

(b) What is the future value of the investment after 9 months?

Ex.2 (#18) How long does it take for $8500 invested at 11% annual simple interest to be worth $13,000?
Ex. 3 (#22) Suppose you lent $5000 to friend 1 for 18 months at an annual simple interest rate of 9%. After 1 year you need money for an emergency and decide to sell note to friend 2.

(a) How much does friend 1 owe when the loan is due?
(b) If your agreement with friend 2 means she earns simple interest at an annual rate of 12%, how much did friend 2 pay you for the note?

Definitions:

- A **sequence function** is a function that has the positive integers as its domain and determines its output uniquely.
- A **sequence** is an ordered list of values (terms) as the output of a sequence function.
- Each value of a sequence is called a **term**, denoted by \( a_1 \) (first term), \( a_2 \) (second term), \( \cdots \).

Ex. 4 (#26) Write the first five terms of the sequence whose \( n \)th term is \( a_n = \left(-\frac{1}{2}\right)^n \).

Definition: An **arithmetic sequence** is a sequence whose terms after the first are given by adding a fixed constant to the previous term. So the \( n \)th term is given by

\[
a_n = a_{n-1} + d = (a_{n-2} + d) + d = \cdots = a_1 + (n - 1)d,
\]

where \( d \) is the common difference.

Ex. 5 (#36) Find the 73rd term of the arithmetic sequence with first term 20 and tenth term 47.
Formula: The sum of the first $n$ terms of an arithmetic sequence is given by

$$s_n = \frac{n}{2}(a_1 + a_n).$$

Ex.6 (#42) Find the sum of the first 200 terms of the arithmetic sequence 12, 9, 6, \ldots

Ex.7 (#46) If you make $27,000 and get $1800 raises each year, in how many years will your salary double?