PROBLEMS

Exercise 1. Consider the function

\[ P(x) = 8x^5 - 18x^4 - x^3 + 18x^2 - 7x. \]

Our goal is to find all critical points and points of inflection.

a. Graph the function. You may have to adjust your axes to make the graph look nice—you should be able to see five points where \( P(x) = 0 \).

b. Have Maple find the derivative of \( P(x) \) with the command \( \text{dP} := \text{D}(P); \). Graph the derivative and indicate all regions where the function \( P(x) \) is increasing.

c. Locate the critical points on your graph. Use the \text{fsolve} command to make Maple solve for the \( x \) value at each critical point.

d. Have Maple find the second derivative of \( P(x) \) (take the derivative of the derivative). Graph it, and indicate all regions where the function \( P(x) \) is concave up.

e. Locate the points of inflection on your graph. Use Maple to solve for the \( x \) value at each point of inflection.

Exercise 2. Consider the function

\[ r(x) = \frac{u(x)}{v(x)} = \frac{1 + x}{2 + x^2 + x^3}. \]

a. Make one graph of \( u(x) \) and \( v(x) \) for \( 0 \leq x \leq 1 \), and another of \( r(x) \). Could you have guessed the shape of \( r(x) \) from looking at the graphs of \( u(x) \) and \( v(x) \)?

b. What happens on the graph of \( u \) and \( v \) at the critical point of \( r \)?

c. Find the exact location of the critical point of \( r \) and set it equal to \( x_r \).

d. Compare \( \frac{u'(x_r)}{u(x_r)} \) with \( \frac{v'(x_r)}{v(x_r)} \). Why are they equal?

Exercise 3. Consider the function

\[ p(x) = u(x)v(x) = (1 + x)(2 + x^2 + x^3) \]

using the same \( u(x) \) and \( v(x) \) as in Exercise 2.

a. Make one graph of \( u(x) \) and \( v(x) \) for \(-2 \leq x \leq 0 \), and another of \( p(x) \). Could you have guessed the shape of \( p(x) \) from looking at the graphs of \( u(x) \) and \( v(x) \)?

b. What happens on the graph of \( u \) and \( v \) at the critical point of \( p \)?

c. Find the exact location of the critical point of \( p \) and set it equal to \( x_p \).

d. Compare \( \frac{u'(x_p)}{u(x_p)} \) with \( \frac{v'(x_p)}{v(x_p)} \). Why are they equal but of opposite sign?