1. We can use Maple to illustrate the solutions of equations with Newton’s method. The lung updating function

\[ c_{t+1} = (1 - q)(c_t - \frac{ac_t}{k + c_t^2}) + q \gamma \]

has an equilibrium, but is virtually impossible to solve for exactly. Define the updating function as the function \( f(c) \) (Maple won’t let you use the letter \( \gamma \) — try something like \( \text{gam} \)). Set \( q := 0.6, \text{gam} := 0.5, k := 1.0 \) and \( \alpha := 0.5 \). The Maple \texttt{solve} command finds the equilibrium without trouble. Try it! But how does Maple do it?

(a) Use the \texttt{cobweb} command to see what this updating function does. How could you use the Intermediate Value Theorem to show that there must be an equilibrium?

(b) Define the new function

\[ > \text{g} := c \rightarrow f(c)-c; \]

The solution of the equation \( g(c) = 0 \) is the equilibrium of the dynamical system. Start from a good guess (like \( \gamma \)), find the tangent line \( \hat{g} \) to the function \( g \) and plot both functions. To find the tangent line, you will need to take the derivative of \( g(c) \). Thanks to Maple’s ornery behavior, you may have to use the following obscure command rather than the simpler \( g \) \( p \) = \( D(g) \):

\[ > \text{gp} := \text{unapply}(\text{diff}(g(c),c),c); \]

(c) Find a scale where you can see both where the two curves intersect the \( x \) axis and the difference between the curves. Use the \texttt{solve} command to solve \( \hat{g}(c) = 0 \). Indicate where this solution appears on your graph.

(d) Find the Newton’s method updating function associated with the function \( g \). Apply it starting with the same initial condition you used in part (c). Does the first result match the answer you found in part (c)?

(e) Apply it several times. How long does it take to converge to the equilibrium for as many decimal places as Maple shows?

(f) How long does it take to converge to 100 decimal places of accuracy? If you called the Newton’s method updating function \texttt{newt}, you can see 100 decimal places after one step with the command

\[ > \text{evalf(newt(gam),100);} \]

2. We will see in class that polynomials can be used to approximate other functions by matching derivatives. The following function happens to be the Taylor polynomial of degree 9 that approximates the function \( \sin(x) \) for \( x \) near 0.

\[ > \text{spoly} := x \rightarrow x - x^3/6 + x^5/120 - x^7/5040 + x^9/362880; \]

(a) Show that the first 9 derivatives of the function match the first 9 derivatives of the function \( \sin(x) \) for \( x = 0 \). You can find the derivatives of \( \sin(x) \) pretty easily by hand, and can use the \( D \) command over and over to evaluate the derivatives of \texttt{spoly}.

(b) Plot both of the functions for a region around \( x = 0 \). Over how large a region does the polynomial provide a good approximation? What happens outside that region?