Picard Iteration

**Definition.** The Picard iterates for the problem
\[ y' = f(t, y), \quad y(0) = A \]
are defined by the formulas
\[ y_0(x) = A, \]
\[ y_n(x) = A + \int_0^x f(t, y_{n-1}(t))dt, \quad n = 1, 2, 3, \ldots \]

**Example.** Find and plot the Picard iterates \( y_0, y_1, y_2 \) for the problem
\[ y' = y^2, \quad y(0) = 1. \]

**Solution:** The exact solution is \( y = \frac{1}{1-t} \), defined on the interval \( 0 \leq t < 1 \). The Maple 6 code which does the plot appears below.

```maple
with(plots):
y0:=1:
T:=(f,x)->y0+eval(int(f(t)^2,t=0..x)):
n:=2:
y:=array(0..n): Y:=array(0..n):
y[0]:=x->y0:
for i from 1 to n do
  y[i]:=unapply(T(y[i-1],x),x):
  Y[i]:=plot(y[i](x),x=0..1):
od:
display([seq(Y[i],i=1..n)]):
seq(eval(y[i]),i=1..n);
```

**Problem 1.** Find and plot the Picard iterates \( y_0 \) through \( y_6 \) for the problem
\[ y' = y^2, \quad y(0) = 5. \]
Compare graphically the convergence of the sequence \( \{y_n\} \) to the limit solution \( y = \frac{5}{(1-5t)} \) and discuss the reason for the finite escape time of \( t = \frac{1}{5} \).

**Problem 2.** Find and plot the Picard iterates \( y_0 \) through \( y_5 \) for the problem
\[ y' = y^4, \quad y(0) = 1. \]
Compare graphically the convergence of the sequence \( \{y_n\} \) to the limit solution \( y = \frac{1}{(1-3t)^{1/3}} \) and discuss the reason for the finite escape time of \( t = \frac{1}{3} \).