Differential Equations 5410
Midterm Exam 1, Fall 2002
Exam Date: Monday, 16 September, 2002

Instructions. The four problems below are take-home, due on the date above. Answer checks are expected. If maple assist is used, then please attach the maple output.

The remaining 20% of the exam is in class, 15 minutes, one problem, of a type similar to one of the problems below. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

Scores

Problem 1. Separable Equations.
Problem 2. Linear Equations.
Problem 3. Planar systems.
Problem 4. Application.
Problem 5. In-class, Sept 16.

Average.

1. (Separable Equations) Solve the separable problem for the implicit and explicit solutions. Distinguish equilibrium and non-equilibrium solutions as needed.

\[ 2y' = x \cos x + \cot x - 4(y + 1)^2(x + \csc x) \cos x. \]

2. (Linear Equations) Solve the linear equation \( 2xy'(x) + 5y(x) = \sqrt{x}e^{-2x}, y(1) = 5. \) Expected details include the factorization method and all integration steps (by hand).

3. (Planar systems) State and prove a result for planar autonomous systems that parallels the first order result: Solutions of \( y' = f(y) \) exist, they are uniquely determined by initial data, and solutions don’t cross, provided \( f \) is of class \( C^1. \)

4. (Application: Torricelli’s law) A water tank has the shape of \( y = x^{6/5} \) revolved around the \( y \)-axis. The depth of the water is 20 feet. After 1 hour and 15 minutes, the drain hole at \((0,0)\) empties the tank to a depth of 12 feet. Find the additional time it takes to empty the tank. Ref: Edwards-Penney, Exercise 1.4-52 and equation 1.4-(24).