**Introduction to Linear Algebra 2270-1**

**Sample Midterm Exam 3 Fall 2003**

In-class Exam Date: Wednesday, November 19, 2003

**Instructions.** Calculators are not allowed. Books and notes are not allowed. Time: 15 minutes. You will be given a variety of problems from which to select. The object is to solve two of them in 15 minutes. The longer ones will be identified as worth two problems.

**5. (Exam 3 in-class)**

(a) Prove \( \mathbf{v}_1, \mathbf{v}_2 \) orthogonal and \( A \) an \( n \times n \) orthogonal matrix implies \( A\mathbf{v}_1, A\mathbf{v}_2 \) orthogonal.

(b) Prove \((\sum_{k=1}^{n} x_k)^2 \leq n \sum_{k=1}^{n} |x_k|^2\).

(c) Prove or disprove: \( A \) orthogonal implies \( A^2 \) orthogonal.

(d) Let \( A \) be \( n \times n \) with eigenpairs \( (\lambda_i, \mathbf{v}_i), 1 \leq i \leq n \). Prove that \((A - \lambda_1 I) \cdots (A - \lambda_n I) = 0\).

(e) Let \( V \) be a subspace of \( \mathbb{R}^n \). Prove that \( V \) and \( V^\perp \) meet only in the zero vector.

(f) Let \( A \) be \( m \times n \). Prove that \( \ker(A) = \{\mathbf{0}\} \) implies \( A^T A \) is invertible.

(g) Let \( V \) be an inner product space. Suppose that \( \mathbf{v} = \sum_{k=1}^{n} c_k \mathbf{v}_k \) and \( \{\mathbf{v}_k\} \) is an orthogonal set. Compute \( c_2 \).

(h) Find a \( 3 \times 3 \) matrix \( A \) such that \( \det(A - \lambda I) = -\lambda^3 + 15\lambda^2 - 3\lambda + 2 \).