3. (Inverse of a matrix) An $n \times n$ matrix $A$ is said to have an inverse $B$ if $AB = BA = I$, where $I$ is the $n \times n$ identity matrix. Prove these facts:

1. If $B_1$ and $B_2$ are inverses of $A$, then $B_1 = B_2$.
2. The inverse of the identity $I$ is $I$.
3. The zero matrix has no inverse.
4. In checking the inverse relation $AB = BA = I$, only one of $AB = I$ or $BA = I$ needs to be verified.

4. (Elementary Matrices) Let $A$ be a $3 \times 3$ matrix and $\vec{b}$ a vector in $\mathbb{R}^3$. Define $C = \text{aug}(A, \vec{b})$. Let matrix $F$ be obtained from $C$ by the following: (a) Swap rows 2 and 3; (b) Add $-1$ times row 3 to row 1; (c) Swap rows 1 and 2; (d) Multiply row 2 by $-5$. Write a matrix multiplication formula for $F$ in terms of $C$ and explicit elementary matrices.

5. (RREF method) Let $a$ and $b$ denote constants and consider the system of equations

$$
\begin{pmatrix}
1 & a+b & a \\
0 & 0 & a \\
1 & a+b & 2a
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} =
\begin{pmatrix}
0 \\
a \\
a
\end{pmatrix}
$$

(1) Determine those values of $a$ and $b$ such that the system has a solution.
(2) For each of the values in (1), solve the system.
(3) For each of the solutions in (2), check the answer.