Introduction to Linear Algebra 2270-1
Midterm Exam 1 Fall 2003
Take-Home Exam Date: Friday, 19 September, 2003
Inclass Exam Date: Tuesday, 23 September, 2003

Instructions. The four take-home problems below are to be submitted at class time at the date marked above. Answer checks are expected. If \texttt{maple} assist is used, then please attach the \texttt{maple} output.
The in-class portion of the exam is the last 15 minutes of class, one problem, of a type similar to either problem 3 or problem 4. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. \textbf{(Elementary matrices)} Let $C$ be the augmented matrix of a system $Ax = b$. Verify that each of the operations \texttt{swap}, \texttt{mult} and \texttt{combo} acting on $C$ produces a matrix answer $F$ of the form $F = EC$, where $E$ is a square elementary matrix, obtained from the identity matrix by suitable modification. In particular, define $E$ and supply a general proof for each of the operations \texttt{swap}, \texttt{mult} and \texttt{combo}.

2. \textbf{(Inverse of a matrix)} An $n \times n$ matrix $A$ is said to have an inverse $B$ if $AB = BA = I$, where $I$ is the $n \times n$ identity matrix. Prove these facts:
   1. If $B_1$ and $B_2$ are inverses of $A$, then $B_1 = B_2$.
   2. The inverse of the identity $I$ is $I$.
   3. The zero matrix has no inverse.
   4. In checking the inverse relation $AB = BA = I$, only one of $AB = I$ or $BA = I$ needs to be verified.
      You may reference a theorem or supply your own proof.

3. \textbf{(Gaussian algorithm)} Solve for $x$, $y$, $z$ in the $3 \times 3$ linear system

\begin{align*}
2x + 2ay + cz &= b \\
3x + ay + 2cz &= 2b \\
5x + 3ay + 3cz &= 3b
\end{align*}

using the Gaussian algorithm, for all constant values of $a$, $b$, $c$. Include all algorithm details and an answer check for each of the three separate cases. Sanity check: $a \neq 0$ is one case, with parametric solution $x = 3b/4 - 3ct_1/4$, $y = -b/(4a) + ct_1/(4a)$, $z = t_1$. The case $a = 0$ has subcases $c \neq 0$ and $c = 0$, for one of which you will report \textit{no solution}.

4. \textbf{(Inverse algorithm)} Determine by \texttt{rref} methods the inverse matrix of

\[
A = \begin{pmatrix}
1 & a & 0 \\
a & 0 & b \\
0 & 1 & 1
\end{pmatrix}.
\]

Please state conditions on $a$, $b$ for when the inverse exists. Show all hand details. Include an answer check, preferably done in \texttt{maple}.

Please attach this exam or a copy to the front of your submitted exam on the due date. Kindly staple the left upper corner and write your name on all pages.