Applied Differential Equations 2250-1 and 2250-2  
Midterm Exam 3, Due classtime 7 April 2003

Instructions. Answer checks are expected. If maple assist is used, then please attach the maple output.

The in-class portion of the exam on April 4 is 15 minutes, one problem, of a type similar to one of the four problems. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. (Variation of Parameters) Show the steps in the solution of

\[ y'' - y = -xe^{-x} \]

by variation of parameters, to obtain the general solution

\[ y = c_1 e^x + c_2 e^{-x} + (1/4)x(1+x)e^{-x}. \]

Use formula (3), page 335.

2. (Undetermined Coefficients) Show the steps in the solution of

\[ y''' - y' = x^2 + xe^{-x} + \sin x \]

by undetermined coefficients, to obtain the general solution

\[ y = c_1 e^x + c_2 e^{-x} + c_3 - (1/3)x^3 - 2x + (1/2)\cos(x) + (1/4)e^{-x}x^2 + (3/4)x e^{-x} + (7/8)e^{-x}. \]

3. (Practical Resonance) Given \[ x'' + 10x' + 50x = 16 \cos(\omega t), \] find
   (a) the derivative of the amplitude \( C(\omega) \),
   (b) the practical resonant frequency \( \omega^* \).

Use formulas on pages 346–347. Show all steps used to obtain the answers.

4. (Spring-mass system) Find a spring-mass equation \( mx'' + cx' + kx = F(t) \) whose general solution is given by

\[ x(t) = c_1 e^{-t} \cos(\sqrt{2}t) + c_2 e^{-t} \sin(\sqrt{2}t) + \sin(2t) + 5 \cos(2t). \]

Hint: Apply superposition to determine \( x_h \) and \( x_p \). Let \( m = 1 \). Choose \( c \) and \( k \) to match \( x_h \). Compute \( F(t) \) from the differential equation with \( x \) replaced by \( x_p \).