Instructions. Choose the exam version based upon your last name, e.g., John Murdock chooses exam version L-S, because M of Murdock is between L and S.

The four problems below are take-home, due on the dates above at class time. Answer checks are expected. If maple assist is used, then please attach the maple output. The remaining 20% of the exam is in class, the last 15 minutes of the hour, one problem, of a type similar to # 3 or 4 below. No books, notes, calculators, computers or outside materials allowed.

1. (Periodic harvesting) The population equation $y' = 4y(5-y)-9\sin(2\pi t/3)$ appears to have a steady-state periodic solution that oscillates about $y = 5$. (a) Apply ideas from the example below to make a computer graphic with 6 solution curves that oscillate about $y = 5$. Submit the plot and the maple code. (b) Find by computer experiment a threshold population size $y_1$ so that $y(0) < y_1$ implies $y(t) = 0$ (population dies out) for some later time $t$, while $y(0) > y_1$ implies $y(t) > 0$ forever and the solution $y(t)$ oscillates about $y = 5$. See Figure 2.5.12, page 128.

# Example. See Figure 12, section 2.5
with(DEtools):
de:=diff(y(t),t)=y(t)*(2-y(t))-4*cos(4*Pi*t):
ic:=[y(0)=1.7],[y(0)=2],[y(0)=2.4],[y(0)=2.8]:
DEplot(de,y(t),t=0..4,y=1..3,[ic],stepsize=0.05);

2. (Jules Verne Problem) Assume a model

$$\frac{d^2 r}{dt^2} = -\frac{Gm_1}{(R_1 + r)^2} + \frac{Gm_2}{(R_2 - R_1 - r)^2}, \quad r(0) = 0, \quad r'(0) = v_0,$$

where $R_2$ is the mean center-to-center distance from the earth to the moon and $R_1$ is the mean radius of the earth. The mass $m_1$ of the earth and $m_2$ of the moon appear, plus the universal gravitation constant $G$. All units are MKS.

(a) Explain why this model takes into account the gravitational attraction of both the moon and the earth.

(b) Calculate the distance $r^*$ at which the projectile has net acceleration zero. Give a symbolic answer and also a numerical answer $\approx 3.39 \times 10^8$ meters.
(c) Conduct a numerical experiment to find the flight time to the moon, when
the launch velocity \( r'(0) \) is 44 m/s faster than the minimal launch velocity \( v_0 = \sqrt{2F(0) - 2F(r^*)} \), \( F(r) = \frac{Gm_1}{R_1 + r} + \frac{Gm_2}{R_2 - R1 - r} \). Use the sample maple code below
to do the experiment.

```
# Group 1
G:=6.6726e-11: m1:=5.975e24: m2:=7.36e22:
R1:=6.378e6: R2:=3.84e8: v0:=1000: T:=210:
d:=diff(r(t),t,t)=-G*m1/(r(t)+R1)^2+G*m2/(R2-R1-r(t))^2:
ic:=r(0)=0,D(r)(0)=v0:
p:=dsolve({de,ic},r(t),type=numeric,method=lsode):
Y:=t->rhs(p(t)[2]):
plot('Y(t)',t=0..T);
```

3. (Gaussian algorithm) Solve for \( x, y, z \) in the \( 3 \times 3 \) linear system

\[
\begin{align*}
2x & + 2(a-b)y + cz = -b \\
1x & + (b-a)y + cz = b \\
3x & + (a-b)y + 2cz = 0
\end{align*}
\]

using the Gaussian algorithm, for all constant values of \( a, b, c \). Include all algorithm
details and an answer check for each of the three separate cases. Sanity check: \( a-b \neq 0 \)
is one case, with parametric solution \( x = b/4 - 3ct_1/4, y = -3b/(4a-4b) + ct_1/(4a-4b), z = t_1 \). The case \( a-b = 0 \) has subcases \( c \neq 0 \) and \( c = 0 \), for one of
which you will report no solution.

4. (Inverse matrix) Determine by \texttt{rref} methods the inverse matrix of

\[
A = \begin{pmatrix} 2 & b & 0 \\ a & 0 & -b \\ 0 & 1 & 1 \end{pmatrix}.
\]

Please state conditions on \( a, b \) for when the inverse exists. Show all hand details.
Prove that in the absence of your condition, no inverse exists. Include an answer
check, preferably done in \texttt{maple}. 

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