Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 1, Fall 2003
Take-Home Exam Date: Wednesday, 17 September, 2003
Inclass Exam Date: Friday, 19 September, 2003

Instructions. The four problems below are take-home, due on the date above at class time. Answer checks are expected. If maple assist is used, then please attach the maple output.

The remaining 20% of the exam is in class, the last 15 minutes of the hour, one problem, of a type similar to one of the problems below. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. (Quadrature Equations) Solve by the method of quadrature the initial value problem $y''' = x^3 - xe^{-4x} - \cos 2x$, $y(0) = y'(0) = 0$, $y''(0) = 1$. Show all integration steps, by hand. An answer check is required.

2. (Separable Equations) Solve the separable problem for equilibrium and non-equilibrium solutions. Identify the implicit solution. Find, if possible, an explicit solution. Check all answers.

\[ 10y' = \sin x + \tan x - 25y^2(1 + \sec x)\sin x. \]

3. (Linear Equations) Solve the linear equation $2xy'(x) + 7y(x) = \sqrt{xe^{-2x}}$, $y(1) = 4$. Expected details include the factorization method and all integration steps, by hand. An answer check is required.

4. (Application: Torricelli’s law) A water tank has the shape of $y = x^{8/7}$ revolved around the $y$-axis. The depth of the water is 19 feet. After 1 hour and 5 minutes, the drain hole at $(0,0)$ empties the tank to a depth of 9 feet. Find the additional time it takes to empty the tank. Ref: Exercise 1.4-52 and equation 1.4-(24). A sane answer is less than 60 minutes.

Please attach this exam or a copy to the front of your submitted exam on the due date. Kindly staple the left upper corner and write your name on all pages. Circle 2250-1 (7:30) or 2250-2 (10:45) at the top of the exam.