Theorem 1 (Peano)

Let \((x_0, y_0)\) be the center of a box

\[
B = \{(x, y) : |x - x_0| \leq H, |y - y_0| \leq K\}
\]

and assume \(f(x, y)\) is continuous on \(B\). Then there is a small \(h > 0\) and a function \(y(x)\) continuously differentiable on \(|x - x_0| < h\) such that \((x, y(x))\) remains in \(B\) for \(|x - x_0| < h\) and \(y(x)\) is one solution (many more might exist) of the initial value problem

\[
y' = f(x, y), \quad y(x_0) = y_0.
\]
Definition 1 (Picard Iteration)
Define the constant function $y_0(x) = y_0$ and then define by iteration

$$y_{n+1}(x) = y_0 + \int_{x_0}^{x} f(t, y_n(t)) dt.$$ 

The sequence $y_0(x), y_1(x), \ldots$ is called the sequence of Picard iterates for $y' = f(x, y), y(x_0) = y_0$. 
Theorem 2 (Picard-Lindelöf)

Let \((x_0, y_0)\) be the center of a box

\[
B = \{(x, y) : |x - x_0| \leq H, |y - y_0| \leq K\}
\]

and assume \(f(x, y)\) and \(f_y(x, y)\) are continuous on \(B\). Then there is a small \(h > 0\) and a unique function \(y(x)\) continuously differentiable on \(|x - x_0| < h\) such that \((x, y(x))\) remains in \(B\) for \(|x - x_0| < h\) and \(y(x)\) solves

\[
y' = f(x, y), \quad y(x_0) = y_0.
\]

The equation

\[
\lim_{n \to \infty} y_n(x) = y(x)
\]

is satisfied for \(|x - x_0| < h\) by the Picard iterates \(y_n\).