Quiz 6

1. Find \( \frac{\partial w}{\partial t} \) by using the chain rule. Express your final answer in terms of \( s \) and \( t \).

\[
\begin{align*}
  w &= x^2y \\
  x &= st \\
  y &= s - t
\end{align*}
\]

\[
\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t}
\]

\[
= (2xy)(s) + (x^2)(-1)
\]

Converting to \( s, t \)

\[
\begin{align*}
  &= 2(st)(s-t)(s) + (st)^2(-1) \\
  &= 2s^2t(s-t) - st^2 \\
  &= 2s^3t - 2st^2 - st^2
\end{align*}
\]

\[
\frac{\partial w}{\partial t} = 2s^3t - 2st^2
\]
\( f(x, y, z) = 2x^2 + 3y^2 - z \)

\( \nabla f \) is normal to the tangent plane

\( 8\hat{i} - 3\hat{j} - \hat{k} \) is normal to given plane \( 8x - 3y - z = 0 \)

since the two planes are parallel,
so the two normal vectors are also parallel

\( \nabla f(x, y, z) \parallel 8\hat{i} - 3\hat{j} - \hat{k} \)

Therefore \( \nabla f(x, y, z) = c(8\hat{i} - 3\hat{j} - \hat{k}) \) for some \( c \) (scalar multiple)

\( \nabla f(x, y, z) = f_x \hat{i} + f_y \hat{j} + f_z \hat{k} \)
\( = 4x \hat{i} + 6y \hat{j} - \hat{k} \)

\( \Rightarrow 4x \hat{i} + 6y \hat{j} - \hat{k} = c(8\hat{i} - 3\hat{j} - \hat{k}) \)

\( \Rightarrow \)
\( 4x = 8c \)
\( 6y = -3c \)
\( -k = -c \quad \Rightarrow \quad c = 1 \)
\( \Rightarrow \quad x = 2 \)
\( \Rightarrow \quad y = -\frac{1}{2} \)

When \( (x, y) = (2, -\frac{1}{2}) \)

\( z \bigg|_{(2, -\frac{1}{2})} = 2(2)^2 + 3\left(-\frac{1}{2}\right)^2 = 8 + \frac{3}{4} = \frac{35}{4} \)

So the required point on the tangent plane is \( (2, -\frac{1}{2}, \frac{35}{4}) \)