Quiz #1 Solutions

(1) Find the general solution of the linear system whose augmented matrix is \[
\begin{bmatrix}
1 & -2 & -1 & 4 \\
-2 & 4 & -5 & 6
\end{bmatrix}.
\] (4 points)

Solution:

\[
\begin{bmatrix}
1 & -2 & -1 & 4 \\
-2 & 4 & -5 & 6
\end{bmatrix} + 2R_1 \rightarrow \begin{bmatrix}
1 & -2 & -1 & 4 \\
0 & 0 & -7 & 14
\end{bmatrix} \cdot \frac{1}{7}
\rightarrow \begin{bmatrix}
1 & -2 & -1 & 4 \\
0 & 0 & 1 & -2
\end{bmatrix} + R_2
\rightarrow \begin{bmatrix}
1 & -2 & 0 & 2 \\
0 & 0 & 1 & -2
\end{bmatrix}
\]

The solution set is \[
\begin{cases}
    x_1 = 2 + 2x_2 \\
    x_2 \text{ is free} \\
    x_3 = -2
\end{cases}
\]

(2) Invent a linear system of two equations in three variables that is inconsistent. (2 points)

Possible solution:

\[
\begin{align*}
x_1 + x_2 + x_3 &= 0 \\
x_1 + x_2 + x_3 &= 1
\end{align*}
\]

Possible solution:

\[
\begin{align*}
0x_1 + 6x_2 - x_3 &= 12 \\
0x_1 + 0x_2 + 0x_3 &= 4
\end{align*}
\]
(3) Give a geometric description of \( \text{Span} \left\{ \begin{bmatrix} 3 \\ -2 \end{bmatrix} \right\} \). (1 point)

Possible solution: The line in \( \mathbb{R}^2 \) passing through the origin and \( \begin{bmatrix} 3 \\ -2 \end{bmatrix} \).

Possible solution: The line in \( \mathbb{R}^2 \) through the origin with slope \(-\frac{2}{3}\).

Possible solution: The line in the plane with equation \( y = -\frac{2}{3}x \).

Description of my favorite solution: Sketch the line in the plane that the other solutions are describing in words.

(4) Bonus problem: Let \( A \) be a 6 \( \times \) 4 matrix (6 rows, 4 columns) viewed as the augmented matrix of a linear system. Assume the linear system is consistent and that \( A \) is in reduced echelon form. Determine how many entries of \( A \) are equal to 0 in each of the following cases:

(a) \( A \) has as many entries equal to 0 as possible.

(b) \( A \) has as few entries equal to 0 as possible.

(1 bonus point)

Solution:

(a) 24 (\( A \) is the 6 \( \times \) 4 matrix of all zeros)

(b) 18, for example

\[
A = \begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\]