Mathematics 2210 PRACTICE EXAM III Fall 2019

1. Evaluate the integral
$$\iint_R (x+y)dA,$$
where R is the triangular region with vertices $(0,0),(0,4)$ and $(1,4)$.

2. Evaluate the iterated integral,
$$\int_0^2 \int_0^{\sqrt{4-x^2}} (x+y)dydx.$$

3. Evaluate the following integral by changing to polar coordinates,
$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} xdxdy.$$

4. Compute the surface area of the bottom part of the paraboloid $z = x^2 + y^2$ that is cut off by the plane $z = 9$.

5. Compute the surface area of the part of the sphere $x^2+y^2+z^2 = a^2$ inside the circular cylinder $x^2 + y^2 = b^2$, where $0 < b \leq a$.

6. Compute the volume of the solid in the first octant bounded by $y = 2x^2$ and $y+4z = 8$.

7. Compute the Jacobian $J(r,\theta)$ of the transformation from polar coordinates to Cartesian coordinates given below:
$$\begin{aligned} x =& r\cos\theta \\ y =& r\sin\theta. \end{aligned}$$

8. Compute the Jacobian $J(x,y)$ of the transformation from Cartesian coordinates to polar coordinates given below:
$$\begin{aligned} r =& \sqrt{x^2+y^2} \\ \theta =& \tan^{-1}\left(\frac{y}{x}\right) \end{aligned}$$
What is the relationship between $J(r,\theta)$ in the previous problem and $J(x,y)$?

9. Let $u(x,y) = \log\sqrt{x^2+y^2} = \log r$.
   (a) Find the vector field associated with this scalar field, by computing $\text{grad}\, u = \nabla u$.
   (b) Compute $\text{curl}(\text{grad}\, u) = \nabla \times (\nabla u)$.
   (c) What are the level sets of $u(x,y)$?

10. Let $\varphi(x, y) = x^2 - y^2$.

    (a) Compute $\vec{F} = -\operatorname{grad}\varphi = -\nabla\varphi$.
    (b) Sketch a diagram in the plane of the vector field $\vec{F}$.
    (c) Compute $\nabla \cdot (\nabla\varphi)$.
    (d) Based on your findings, what kind of function is φ?

11. Let $\varphi(x, y) = x^2 + y^2$.

    (a) Compute $\vec{F} = -\operatorname{grad}\varphi = -\nabla\varphi$.
    (b) Sketch a diagram in the plane of the vector field $\vec{F}$.
    (c) Compute $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$.

12. Find $\operatorname{div}\mathbf{F}$ and $\operatorname{curl}\mathbf{F}$, where $\mathbf{F}(x, y, z) = x^2\mathbf{i} - 2xy\mathbf{j} + yz^2\mathbf{k}$.

13. Find the volume of a spherical ball of radius a using a triple integral.

14. Find the mass of a cylinder of radius a and height h if its mass density is proportional to the distance to its base.