Mathematics 1210 **PRACTICE EXAM III** **Fall 2018**

1. Calculate the following integrals:

   (a) $\displaystyle\int_0^4 \sqrt{x}\,dx$ (b) $\displaystyle\int_0^{\pi/2} \sin x\,dx$ (c) $\displaystyle\int_1^3 \frac{1-3x^3}{x^2}\,dx$

   (d) $\displaystyle\int_0^{\pi} \sin^2 x\,dx$ (e) $\displaystyle\int_0^{\pi} x\cos(x^2+\pi)\,dx$ (f) $\displaystyle\int_{-3}^{3} x^3\,dx$

2. Find the general solution to each of the following differential equations.

   (a) $\displaystyle\frac{dy}{dx} = \sqrt[3]{\frac{x}{y}}$ (b) $\displaystyle\frac{d^2x}{dt^2} = -\omega^2 x$ (c) $\displaystyle\frac{d^2x}{dt^2} = -g$

3. Consider the function

$$f(x) = \sqrt{x}\,.$$

   (a) Find the average slope of this function on the interval $(1,4)$.
   (b) By the Mean Value Theorem, we know there exists a c in the open interval $(1,4)$ such that $f'(c)$ is equal to this mean slope. What is the value of c in the interval which works?

4. A population of rabbits in a forest is found to grow at a rate proportional to the cube root of the population size P. The initial population is 1000 rabbits, and 5 years later there are 1728 of them.

   (a) Write the differential equation for the rabbit population $P(t)$ along with the two corresponding conditions.
   (b) Solve this differential equation, that is, find the particular solution which incorporates both conditions.
   (c) How long does it take for the rabbit population to quadruple (reach 4000) from its initial value of 1000?

5. Calculate $\displaystyle\int_1^2 (3x^2-2)\,dx$ from the definition of the integral, that is, using Riemann sums. (Hint: Use $x_i = 1 + \frac{i}{n}$ and $\Delta x = \frac{1}{n}$.) Check your result using the Fundamental Theorem of Calculus.

6. Newton's second law for the position $x(t)$ of an object in Earth's gravitational field is

$$F = m\frac{d^2x}{dt^2},$$

   where $F = -mg$, m is the object's mass and $g = 32$ ft/s^2 is the acceleration due to the Earth's gravity.

   (a) Find $x(t)$ that satisfies this second order linear differential equation and the initial conditions $x(0) = x_0$ feet and $v(0) = v_0$ f/s. (Hint: First solve

$$\frac{dv}{dt} = -g,$$

   where $v(0) = v_0$ and $v = \dfrac{dx}{dt}$.)

(a) An object is thrown downward from a height of 64 feet with velocity $v_0 = -10$ f/s. How long does it take for the object to hit the ground? (Hint: Use your result from part (a)).

7. Find the following: (a) $\dfrac{d}{dx} \displaystyle\int_0^{x^2} \tan\theta \, d\theta$ (b) $\displaystyle\lim_{x\to 0} \frac{\int_0^x (1-\cos t)\, dt}{x^3}$

8. Calculate the following:

(a) $\displaystyle\sum_{k=1}^{10} (2^k - 2^{k+1})$ (b) $\displaystyle\sum_{k=1}^{100} (2k^2 + 1)$