Mathematics 1210 PRACTICE EXAM II Fall 2018

1. Calculate the following:

   (a) $\dfrac{d^2x}{dt^2}$, $x(t) = A\sin(\omega t - \phi)$ (b) $\dfrac{df}{dx}$, $f(x) = \left(\dfrac{x-2}{x-\pi}\right)^3$

   (c) $\dfrac{dy}{dx}$, $\cos xy = y^2 + 2x$ (d) $\lim\limits_{x\to 0} \dfrac{\sin x \tan x}{1-\cos x}$

   (e) $\dfrac{df}{dx}$, $f(x) = \sin\sqrt{\dfrac{\tan x}{1+x^2}}$ (f) $\dfrac{df}{dx}$, $f(x) = x^2 \sin^2(x^3)$

   (g) $\dfrac{dm}{dv}$, $m(v) = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$, m_0 is rest mass, and $c = 3.0 \times 10^8$ m/s.

   (h) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x}$ (i) $\lim\limits_{h\to 0} \dfrac{(x+h)^3 - x^3}{h}$ (j) $\lim\limits_{x\to 0} \dfrac{\sin x - x}{x^3}$

2. A circular oil slick spreads so that its radius increases at the rate of 1.5 feet/second. How fast is the area of the enclosed oil increasing at the end of two hours?

3. A balloon initially on the ground 150 feet away from an observer is released and rises at a rate of 8 f/s. How fast is the distance between the observer and the balloon changing when the balloon is 50 feet above the ground ?

4. Approximate $\sqrt{66}$ and $\sin\left(\dfrac{\pi}{100}\right)$ using linear approximation (i.e., the differential).

5. Use the differential to approximate the increase in volume of a spherical bubble as its radius increases from 3 to 3.025 inches.

6. Consider the following functions $f(x)$. In each case, find all local maxima and minima of f, where f is increasing and decreasing, where f is concave up and concave down, and all inflection points. Does f have a global maximum or a global minimum? Sketch the graph of $f(x)$.

   (a) $f(x) = x^3 - 12x + 1$

   (b) $f(x) = \dfrac{1}{1+x^2}$

7. Consider $f(x) = 2\sin\left(x - \frac{\pi}{4}\right)$ on the interval $[\frac{\pi}{2}, \frac{5\pi}{4}]$. Find where f is increasing and decreasing. Find the maximum and minimum values of f on the interval.

8. A rectangle has two corners on the $x-$axis and the other two on the parabola $y = 12 - x^2$, with $y \geq 0$. What are the dimensions of the rectangle of this type with maximum area?

9. An object is propelled upward from the ground with initial velocity $v_0 = 32$ m/s. After time t, the height $x(t)$ of the object is given by $x(t) = -16t^2 + v_0 t + x_0$, where x_0 is the initial position, which is assumed to be $x_0 = 0$.

   (a) What is the velocity of the object when it reaches its maximum height?

   (b) At what time does the object reach its maximum height?

   (c) What is the maximum height reached by the object?