Problem 1 Assume that Archie, Betty and Veronica are in Walgreen’s to buy Halloween candy.

(a) There are two cashier’s available that serve in average one person per two minutes. If the trio approaches the registers in alphabetical order, what is the expected time in minutes that Veronica can go to the register herself?
(b) What is the probability that Veronica will go to Archie’s teller?
(c) What is the probability that Veronica will wait 7 minutes until a register is free?
(d) What is the expected total amount of time (in minutes) for Veronica to exit the store?
(e) What is the probability that Veronica is the last to leave? (Careful this is very hard to compute)

Problem 2 Let \(X_t\) and \(Y_t\) two independent Poisson processes with arrival rates \(\mu\) and \(\lambda\) respectively, that model the arrivals of customers at stores Xanadu and Yliaster (again respectively).

(1) What is the probability that a customer visits Xanadu before a customer visits Yliaster?
(2) What is the probability that in the first two hours a total of 7 customers arrived at the two stores. For this part, assume that time is measured in quarters of an hour (so we expect \(\mu\) customers per 15 minutes in Xanadu.)
(3) Given that exactly 7 customers arrived at the two stores, what is the probability that they all went to Yliaster?
(4) Assume \(T\) is the time when the first customer arrived in Xanadu. Find \(P\{Y_T = k\}\), i.e. the probability that \(k\) customers arrived in Yliaster by the time one customer arrived in Xanadu.

Problem 3

(1) Let \(A\) be the infinitesimal generator of a finite-state, continuous time MC. Assume that the largest entry of \(A\) in absolute value is \(M\). Define
\[
P = (2M)^{-1}A + I.
\]
- Show that \(P\) can be interpreted as a transition matrix for a discrete time, irreducible, aperiodic Markov Chain.
- Using the above, show that \(A\) has a unique left 0- eigenvector that is a probability vector.
(2) If \(A = QDQ^{-1}\) for some diagonal matrix \(D\), show that for all \(n \in \mathbb{N}\),
\[
A = QD^nQ^{-1}
\]
Problem 4 You may use a computer for the matrix algebra. Do not use a computer for anything else.

(1) Problem 3.5, p. 84 from your book.
(2) Problem 3.9, p. 84 from your book

Problem 5 Do problem 3.1, p.82 from your book.