Intro to Graphs: (1-12)

1) [Diagram] not a graph of a function, fails the vertical line test

2) [Diagram] Is a graph of a function, passes the vertical line test

3) [Diagram] Not a graph of a function, passes the vertical line test

4) [Diagram] Is a graph of a function, passes the vertical line test
5) \[ X\text{-ints: } (-3,0), (2,0), (4,0) \]
\[ y\text{-ints: } (0,2) \]

6) \[ X\text{-ints: } (4,0) \]
\[ y\text{-ints: } (0,-2) \]

7) \[ \text{Domain } = (1, 3] \]
\[ \text{Range } = (1, 4] \]

8) \[ \text{Domain } = [2, 4] \]
\[ \text{Range } = [1, 2] \]
9) \( Domain = [-2, 3) \)  
   Range = \([-5, 4]\)

10) \( Domain = (1, 5) \)  
    Range = \((-1, 4]\)

11) \( Domain = \mathbb{R} \)  
    Range = \(\mathbb{R}\)

12) \( Domain = \mathbb{R} \)  
    Range = \([-2, \infty)\)
Graph Transformations: 1-16

1) \( f(x) + 2 = x^8 + 2 \)  \( \text{D} \)

2) \( 3f(x) = 3x^8 = \)  \( \text{I} \)

3) \( f(-x) = (-x)^8 = x^8 \)  \( \text{A} \)

4) \( f(x-2) = (x-2)^8 \)  \( \text{G} \)

5) \( \frac{1}{3} f(x) = \frac{1}{3} x^8 \)  \( \text{B} \)

6) \( f(3x) = (3x)^8 \)  \( \text{H} \)

7) \( f(x)-2 = x^8-2 \)  \( \text{C} \)

8) \( -f(x) = -x^8 \)  \( \text{F} \)

9) \( f(x+2) = (x+2)^8 \)  \( \text{J} \)

10) \( f\left(\frac{x}{3}\right) = \left(\frac{x}{3}\right)^8 \)  \( \text{E} \)

11) \(-4g(3x-7)+2 = -4\left(\frac{1}{3x-7}\right)+2 \)
   \( = -\frac{4}{3x-7}+2 \)  \( \text{B} \)

12) \( 6g\left(-\frac{1}{2x+5}\right)-3 = 6\left(\frac{1}{-2x+5}\right)-3 \)
   \( = \frac{6}{-2x+5}-3 \)  \( \text{A} \)
13) After transformation of $\uparrow 2$
\[ f(x+2) \]
\[ \Rightarrow (4,1) \rightarrow (4,3) \quad \text{A} \]

14) After transformation of $\downarrow 2$
\[ f(x)-2 \]
\[ (4,1) \rightarrow (4,-1) \quad \text{D} \]

15) $f(x+2)$ Before transformation of $\leftarrow 2$
\[ (4,1) \rightarrow (2,1) \quad \text{B} \]

16) $f(x-2)$ Before transformation of $\rightarrow 2$
\[ (4,1) \rightarrow (6,1) \quad \text{C} \]

Inverse Functions: (1-3, 7-9, 15-20)

1) $g(2) = 3$ therefore $g^{-1}(3) = 2$

2) $g(7) = -2$ therefore $g^{-1}(-2) = 7$

3) $g(-10) = 5$ therefore $g^{-1}(5) = -10$

Using the chart: ( $f^{-1}(5) = -7$, $f^{-1}(3) = 2$, $f^{-1}(1) = -2$ )

7) $f(x+2) = 5$
\[ \Rightarrow f^{-1}(5) = (x+2) = -7 \]
\[ x = -9 \]
8) \[ f(3x - 4) = 3 \]
\[ \Rightarrow 3x - 4 = f^{-1}(3) = 2 \]
\[ 3x = 6 \quad \boxed{x = 2} \]

9) \[ f(-5x) = 1 \]
\[ \Rightarrow -5x = f^{-1}(1) = -2 \]
\[ x = \frac{-2}{-5} \quad \Rightarrow \boxed{x = \frac{2}{5}} \]

15) \[ h(x) = \frac{1}{x} \]
\[ \Rightarrow y = \frac{1}{x} \]
\[ xy = 1 \]
\[ x = \frac{1}{y} = -f^{-1}(y) \quad \Rightarrow \boxed{f^{-1}(x) = \frac{1}{x}} \]

16) \[ f(x) = \frac{x}{x - 1} \]
\[ \Rightarrow y = \frac{x}{x - 1} \]
\[ y(x - 1) = x \]
\[ xy - y = x \]
\[ -y = x - xy \]
\[ -y = x(1-y) \]
\[ \Rightarrow f^{-1}(x) = \frac{-x}{1-x} = \frac{x}{x-1} \]
17) \[ g(x) = \frac{2x + 3}{x} \]

\[ \Rightarrow y = \frac{2x + 3}{x} \]
\[ xy = 2x + 3 \]
\[ xy - 2x = 3 \]
\[ x(y - 2) = 3 \]
\[ x = \frac{3}{y - 2} = f^{-1}(y) \]

18) \[ h(x) = \frac{x}{4-x} \]

\[ \Rightarrow y = \frac{x}{4-x} \]
\[ y(4-x) = x \]
\[ 4y - xy = x \]
\[ 4y = x + xy \]
\[ 4y = x(1+y) \]
\[ \frac{4y}{1+y} = x = h^{-1}(y) \]

\[ h^{-1}(x) = \frac{4x}{1+x} \]
19) \( f : \mathbb{R} \rightarrow (0, \infty) \)

- \( f \) passes the horizontal line test \( \Rightarrow f \) is one-to-one
- \( \text{Range} = (0, \infty) = \text{target} \)
- \( \Rightarrow f \) is onto

Since \( f \) is both one-to-one and onto, \( f \) has an inverse.

20) \( g : (0, \infty) \rightarrow \mathbb{R} \). Does \( g \) have an inverse?

- \( g \) passes the horizontal line test \( \Rightarrow g \) is one-to-one
- \( \text{Range} = \mathbb{R} = \text{target} \) therefore \( g \) is onto

\( g \) is one-to-one and onto, \( g \) has an inverse.
Other Problems:

1) A graph of a function can have infinitely many x-intercepts (consider \( f(x) = 0 \)).

A graph of a function may only have at most one y-intercept (if it had more than one it would fail the vertical line test).

2) A graph may fail the vertical line test; a graph of a function may not.