Math 1050-006 Homework 1 Solutions

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1 Book Problems

1.1 Sets and Numbers

1. True

2. False, $4 \notin \{14, 44, 43, 24\}$

3. False, $\mathbb{Z} = \{-\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, $\frac{1}{3} \notin \mathbb{Z}$

4. False, $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$, $-5 \notin \mathbb{N}$

5. True

6. True

13. False, $-2 \notin \mathbb{N}$, therefore $\{-2, 3, 0\} \not\subseteq \mathbb{N}$

14. True

15. True

16. False, $\sqrt{2} \notin \mathbb{Q}$, therefore $\{\sqrt{2}, 271\} \not\subseteq \mathbb{Q}$

1.2 Rules for Numbers

1. True

2. False, $3(x + y) = 3x + 3y \neq 3x + y$

6. False, $2 \in [2, 5]$, but $2 \notin (2, 5]$

7. True

11. True

12. False, $-17 \in [-17, \infty)$, but $-17 \notin (-17, \infty)$, therefore $[-17, \infty) \not\subseteq (-17, \infty)$.

16. False, $10 \notin (2, 8)$, therefore $\{3, 10, 7\} \not\subseteq (2, 8)$

17. True
1.3 Functions
1. \( f(5) = \frac{1}{5^2} = \frac{1}{25} \)
2. \( f(10) = \frac{1}{10^2} = \frac{1}{100} \)
3. \( g(1) = \frac{2(1)^2-(1)+1}{(1)^2+1} = \frac{2-1+1}{1+1} = 1 \)
4. \( g(-3) = \frac{2(-3)^2-(-3)+1}{(-3)^2+1} = \frac{18+3+1}{9+1} = \frac{22}{10} = 2.2 \)
5. \( h(0) = 14 \) (constant function \( h(x)=14 \))
6. \( h(\frac{\pi}{6}) = 14 \) (constant function \( h(x)=14 \))
7. \( id(15) = 15 \) (identity function)
8. \( id(-4) = -4 \) (identity function)

1.4 Sequences
1. Neither, difference between 2 and 7 is 5, but the difference between 7 and 14 is 7, therefore not an arithmetic sequence. Also we need to multiply 2 by \( \frac{7}{2} \) to get 7, but we need to multiply 7 by 2 to get 14, so not a geometric sequence.

2. Arithmetic, add 4 to each term to get the next term.

3. Geometric, multiply each term by -1 to get the next term

7. \( a_1 = -1, d = 5 \)
8. \( a_1 = 2, d = -12 \)
13. \( a_1 = -5, r = 5 \)
14. \( a_1 = 4, r = -2 \)
16. 5,7,9,11,... is an arithmetic sequence with \( a_1 = 5, d = 2 \), using the prediction equation for arithmetic sequences \( a_n = a_1 + (n-1)d \) we get that \( a_{4223} = a_1 + (4223-1)(d) = 5 + (4222)(2) = 5 + 8444 = 8449 \)
17. 4,1,-2,-5,... is an arithmetic sequence with \( a_1 = 4, d = -3 \) Therefore \( a_{5224} = a_1 + (5224-1)d = 4 + (5223)(-3) = 4 - 15669 = -15665 \)
18. 54, 18, 6, 2, ... is a geometric sequence with \( a_1 = 54, r = \frac{1}{3} \), using the prediction equation for geometric sequences \( (a_n = r^{(n-1)}a_1) \) we get that
\[
a_7 = r^{7-1}a_1 = \left(\frac{1}{3}\right)^6(54) = \frac{54}{3^6} = \frac{2}{3^3} = \frac{2}{27}
\]

19. -11, 22, -44, 88, ... is a geometric sequence with \( a_1 = -11, r = -2 \).
Therefore \( a_6 = (r)^{(6-1)}a_1 = (-2)^5(-11) = (-32)(-11) = 352 \)

23. \( c_n = (3 - n)(n + 2) \), therefore if we let \( n = 8 \) we get,
\[ c_8 = (3 - 8)(8 + 2) = (-5)(10) = -50 \]

1.5 Sums and Series

2. 3 + 3 + 3 + ... + 3 + 3 (50 times) = 3(50) = 150

4. \((-2) + (-2) + (-2) + (-2) + ... + (-2) + (-2)\) (78 times) = \((-2)(78) = -156\)

5. 1 + 2 + 3 + 4 + 5 + ... + 38 + 39 + 40 is the finite sum of the first 40 terms of an arithmetic sequence which has equation \( \frac{n}{2}(a_1 + a_n) \). Here \( a_1 = 1, n = 40, a_n = 40 \). Therefore
\[
\frac{n}{2}(a_1 + a_n) = \frac{40}{2}(1 + 40) = (20)(41) = 820
\]

8. \((2(1)-1) + (2(2)-1) + (2(3)-1) + (2(4)-1) + (2(5)-1) = 1 + 3 + 5 + 7 + 9 = 25\)

13. Sum of the first 80 terms of the sequence 53, 54, 55, 56, 57, ..., First note that this sequence is an arithmetic sequence with \( a_1 = 53, d = 1 \). We can therefore use the arithmetic sequence sum equation
\[
\left(\frac{n}{2}(a_1 + a_n)\right) \text{ However first we need to solve for } a_n \text{ which we can do using the arithmetic sequence prediction equation } (a_n = a_1 + (n - 1)d).
\]
As \( n = 80 \), we have \( a_{80} = a_1 + (80 - 1)d = 53 + (79)(1) = 132 \) Therefore
\[
\frac{n}{2}(a_1 + a_n) = \frac{80}{2}(53 + 132) = (40)(185) = 7400
\]

16. Sum all of the terms in the geometric sequence 7, \( \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, \ldots \) ... This is a geometric sequence with \( a_1 = 7, r = \frac{2}{3} \), Therefore we can use the sum of a geometric sequence formula, \( \frac{a_1}{1-r} \) Which for this sequence we get:
\[
\frac{7}{1-\frac{2}{3}} = \frac{7}{\frac{1}{3}} = (7)(3) = 21
\]

17. Sum all of the terms in the geometric sequence 25, 15, 9, \( \frac{27}{5} \), ... This is a geometric sequence with \( a_1 = 25, r = \frac{3}{5} \).
\[
\text{Therefore } \frac{a_1}{1-r} = \frac{25}{1-\frac{3}{5}} = \frac{25}{\frac{2}{5}} = (25)(\frac{5}{2}) = \frac{125}{2}
\]
2 Other Problems

1. The difference between an infinite set and a sequence is that order matters in a sequence, but not in an infinite set.

2. \(g(x) = x\) is not an identity function. To be an identity function the Domain and the Target need to be the same, but for \(g(x)\), the Domain=\(\mathbb{N}\) and the Target=\(\mathbb{R}\). As \(\mathbb{N} \neq \mathbb{R}\) we have that \(g(x)\) is not an identity function.

3. A sequence can be both an arithmetic and a geometric sequence, for example consider the sequence 1,1,1,1,1,.... which is an arithmetic sequence with \(a_1 = 1, d = 0\) and a geometric sequence with \(a_1 = 1, r = 1\)

4. The fibonacci sequence 1,1,2,3,5,8,13,21,... is neither an arithmetic sequence nor a geometric sequence. Notice that the difference subsequent terms in the sequence is 0,1,1,2,3,5,8,... is not constant, therefore not arithmetic. Furthermore we would have to multiply 1 by 2 to get 2, but multiply 2 by \(\frac{3}{2}\) to get, so \(r\) is not constant and therefore the fibonacci sequence is also not geometric.