Final Review

Discrete Math

For #1-5 find the specific term:

1) Find the 18th term for: 67, 61, 55, ...

2) Find the 21st term for: −14, −10, −6, ...

3) Find the 83rd term for: 16, 8, 4, ...

4) Find the 57th term for: −3, 6, −12, 24, ...

5) Find the 32nd term for: $\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, ...$

For #6-12 find the given sum:

6) $\sum_{i=1}^{4} (i^2 - 1)$
7) \[ \sum_{i=1}^{14} (2 - 5i) \]

8) \[ \sum_{i=1}^{\infty} \left( \frac{4}{3^i} \right) \]

9) \[ \sum_{i=1}^{\infty} \left( \frac{3}{2} \right)^{i-1} \]

10) The sum of the first 18 terms of \(-4 + -1 + 2 + \ldots\)

11) The sum of the first 23 terms of \(81 + 72 + 63 + \ldots\)

12) The infinite sum of \(8 + 4 + 2 + \ldots\)
For #13-21 find the following:

13) You’re at Café Rio and want to order a burrito. There are three kinds of beans, two types of rice, and four kinds of meat fillings for the burrito. How many different kinds of burritos can you make if you select one of each? (You don’t need to simplify)

14) Write \( \binom{9}{3} \) as an integer.

15) Suppose a set A contains 243 objects. How many 92 object subsets of A are there? (You don’t need to simplify)

16) How many ways are there to choose and order 49 objects from a collection of 304 objects? (You don’t need to simplify)

17) How many different 4 letter words can you make with the letters MATH if you use each letter once? (You don’t need to simplify)

18) How many different ways can you order 21 different objects? (You don’t need to simplify)

19) There are 20 people competing for 3 scholarships each worth $5000 per year. How many different ways can you select three different people? (You don’t need to simplify)

20) There are 20 people competing for a $1000, $3000, and $5000 scholarship. How many different ways can you select the three different people? (You don’t need to simplify)
21) How many 3 people committees can you make from 12 people? (You don’t need to simplify)

Graphing

Graph and state the x-intercept(s), y-intercept, domain (D), range (R), leading term (LT), end behavior (EB), or vertical asymptote (VA) when specified:

22) Graph \( f(x) = 2 \)

\[
\begin{array}{c}
\text{x-int(s)} = \\
\text{y-int} = \\
\text{D} = \\
\text{R} = \\
\end{array}
\]

23) Graph \( f(x) = x \)

\[
\begin{array}{c}
\text{x-int(s)} = \\
\text{y-int} = \\
\text{D} = \\
\text{R} = \\
\end{array}
\]

24) Graph \( f(x) = x^2 \)

\[
\begin{array}{c}
\text{x-int(s)} = \\
\text{y-int} = \\
\text{D} = \\
\text{R} = \\
\end{array}
\]

25) Graph \( f(x) = x^3 \)

\[
\begin{array}{c}
\text{x-int(s)} = \\
\text{y-int} = \\
\text{D} = \\
\text{R} = \\
\end{array}
\]
26) Graph \( f(x) = \sqrt[3]{x} \) or any even root function

\[ f(x) = \sqrt[3]{x} \]

x-int(s)=
y-int=
D=
R=

27) Graph \( f(x) = \sqrt[3]{x} \) or any odd root function

\[ f(x) = \sqrt[3]{x} \]

x-int(s)=
y-int=
D=
R=

28) Graph \( f(x) = \frac{1}{x} \)

\[ f(x) = \frac{1}{x} \]

x-int(s)=
y-int=
D=
R=

29) Graph \( f(x) = \frac{1}{x^2} \)

\[ f(x) = \frac{1}{x^2} \]

x-int(s)=
y-int=
D=
R=

30) Graph \( f(x) = a^x \) where \( a > 1 \)

\[ f(x) = a^x \]

x-int(s)=
y-int=
D=
R=

31) Graph \( f(x) = \log_a(x) \) where \( a > 1 \)

\[ f(x) = \log_a(x) \]

x-int(s)=
y-int=
D=
R=
32) Graph $f : (-2, 0] \rightarrow \mathbb{R}$ where $f(x) = x^2$  

\[
f(x) = x^2
\]

D=_________ R=_________

33) Graph $g : \{-4, -2, 2\} \rightarrow \mathbb{R}$ where $g(x) = \frac{1}{2}x - 1$  

\[
g(x) = \frac{1}{2}x - 1
\]

D=_________ R=_________

34) Graph $g(x) = -\log_e(x - 1)$  

\[
g(x) = -\log_e(x - 1)
\]

x-int(s)=_________ y-int=_________

D=_________ R=_________

35) Graph $f(x) = 2e^{-x}$  

\[
f(x) = 2e^{-x}
\]

x-int(s)=_________ y-int=_________

D=_________ R=_________
36) Graph \( n(x) = -\sqrt[3]{x + 1} \)

37) Graph \( h(x) = -\sqrt{x + 1} + 2 \)

38) Graph \( m(x) = 4(x + 2)^2 + 1 \)

39) Graph \( p(x) = -2(x+1)(x+1)(x-2)(x^2+1) \)
40) Graph \( r(x) = \frac{-3(x - 1)(x - 1)}{4(x + 2)(x^2 + 1)} \)

41) Graph

\[ f(x) = \begin{cases} 
1 & \text{if } x \in (-\infty, 1) \\
3 & \text{if } x = 1 \\
x^2 & \text{if } x \in (1, 2] 
\end{cases} \]

42) State the x and y intercept for:

\[ f(x) = \begin{cases} 
e^x & \text{if } x \leq 2 \\
x^2 - 9 & \text{if } x > 2 
\end{cases} \]

43) Write \( \left( \frac{16}{81} \right)^{\frac{2}{3}} \) as a rational number.

**Algebra**
44) Write \(7^{-4} \sqrt[717]{7^{1/3}}\) as an integer.

45) Write \(\log_3 \left( \sqrt[3]{3} \right)\) as a rational number.

46) Write \(7^0\) as an integer.

47) Write \(\log_7(1)\) as an integer.

48) Find \(x\) where \(x^3 \left( \frac{1}{2}x + 3 \right)^3 = 8\)

49) Find \(x\) where \(2 \left( \frac{x^2}{2^x+2} \right) + 5 = 7\)
50) Find $x$ where $(2^{3x-5}) (3) = \frac{5}{2^{1-x}}$

51) Find $x$ where $\log_2(x) - \log_2(1 - x) = 3$

52) Find $x$ where $4 \log_e(x) + \log_e(x^3) + 8 = 11$

53) Find $x$ where $2 = 3 \log_8(x - 1)$

54) Find $g \circ f(x)$ if $f(x) = x + 3$ and $g(x) = x^2$

55) Find the inverse of $g(x) = 2\sqrt[3]{4 - x}$
56) Find the inverse of \( m(x) = 3 \log_e(5 - 2x) \)

57) Find the implied domain for \( p(x) = \sqrt[3]{4 - x} \)

58) Find the implied domain for \( a(x) = \frac{x + 1}{x^2 - 4} \)

59) Find the implied domain of \( f(x) = x^2 - 3x + \log_e(4 - 5x) \)

60) Find the implied domain of \( g(x) = \frac{x^3}{2} - 7\sqrt{x - 4} \)

61) Complete the square: write \( 2x^2 - 4x - 3 \) in the form of \( \alpha(x + \beta)^2 + \gamma \) where \( \alpha, \beta, \gamma \in \mathbb{R} \).
62) How many roots does $2x^2 - 3x + 4$ have?

63) Find the roots of $x^2 - 5x + 2$

64) Find a root of $x^3 + 2x^2 - x + 6$

65) Completely factor: $f(x) = x^3 + 2x^2 - x + 6$ (Hint: $-3$ is a root)

66) Solve for $x$ if $|2x - 3| + 2 < 10$
67) Solve for $x$ if $|3x - 2| - 5 \geq 1$

**Linear Algebra**

68) Find the determinant of the matrix: \[
\begin{pmatrix}
-2 & -3 \\
1 & 0
\end{pmatrix}
\]

69) Find the determinant of the matrix: \[
\begin{pmatrix}
-2 & -1 & 1 \\
0 & -1 & 2 \\
-1 & 0 & 3
\end{pmatrix}
\]

70) Find the product of \[
\begin{pmatrix}
1 & 0 \\
3 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 2 \\
0 & 1
\end{pmatrix}
\]

71) Find the inverse of \[
\begin{pmatrix}
1 & 4 \\
2 & 3
\end{pmatrix}
\]
72) Find \( \begin{pmatrix} -1 & 2 \\ -3 & 4 \end{pmatrix}^{-1} \)

73) Find \( \det \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix} \)

74) Write the following system of three linear equations in three variables as a matrix equation:

\[
\begin{align*}
2x - y + z &= 2 \\
y + 2x &= 1 \\
-x + y - z &= 0
\end{align*}
\]

75) Solve for \( x, y, z \) if:

\[
\begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -1 & 2 & -1 \\ -2 & 2 & -1 \\ 3 & -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ -4 & 5 & 2 \end{pmatrix} \]