Sets & Numbers

Sets

A set is a collection of objects. For example, the set of days of the week is a set that contains 7 objects: Mon., Tue., Wed., Thur., Fri., Sat., and Sun..

Set notation. Writing \{2, 3, 5\} is a shorthand for the set that contains the numbers 2, 3, and 5, and no objects other than 2, 3, and 5.

The order in which the objects of a set are written doesn’t matter. For example, \{5, 2, 3\} and \{2, 3, 5\} are the same set. Alternatively, the previous sentence could be written as “For example, \{5, 2, 3\} = \{2, 3, 5\}.”

If \(B\) is a set, and \(x\) is an object contained in \(B\), we write \(x \in B\). If \(x\) is not contained in \(B\) then we write \(x \notin B\).

Examples.

- \(5 \in \{2, 3, 5\}\)
- \(1 \notin \{2, 3, 5\}\)

Subsets. One set is a subset of another set if every object in the first set is an object of the second set as well. The set of weekdays is a subset of the set of days of the week, since every weekday is a day of the week.

A more succinct way to express the concept of a subset is as follows:

\[
\text{The set } B \text{ is a subset of the set } C \text{ if every } b \in B \text{ is also contained in } C.
\]

Writing \(B \subseteq C\) is a shorthand for writing “\(B\) is a subset of \(C\)”. Writing \(B \not\subseteq C\) is a shorthand for writing “\(B\) is not a subset of \(C\)”.

Examples.

- \(\{2, 3\} \subseteq \{2, 3, 5\}\)
- \(\{2, 3, 5\} \not\subseteq \{3, 5, 7\}\)

Set minus. If \(A\) and \(B\) are sets, we can create a new set named \(A - B\) (spoken as “\(A\) minus \(B\)”) by starting with the set \(A\) and removing all of the objects from \(A\) that are also contained in the set \(B\).
Examples.

• \( \{1, 7, 8\} - \{7\} = \{1, 8\} \)
• \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9\} \)

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Numbers

Among the most common sets appearing in math are sets of numbers. There are many different kinds of numbers. Below is a list of those that are most important for this course.

Natural numbers. \( \mathbb{N} = \{1, 2, 3, 4, \ldots\} \)

Integers. \( \mathbb{Z} = \{..., -2, -1, 0, 1, 2, 3, \ldots\} \)

Rational numbers. \( \mathbb{Q} \) is the set of fractions of integers. That is, the numbers contained in \( \mathbb{Q} \) are exactly those of the form \( \frac{n}{m} \) where \( n \) and \( m \) are integers and \( m \neq 0 \).

For example, \( \frac{1}{3} \in \mathbb{Q} \) and \( \frac{-7}{12} \in \mathbb{Q} \).

Real numbers. \( \mathbb{R} \) is the set of numbers that can be used to measure a distance, or the negative of a number used to measure a distance. The set of real numbers can be drawn as a line called “the number line”.

\[ \sqrt{2} \text{ and } \pi \text{ are two of very many real numbers that are not rational numbers.} \]

(Aside: the definition of \( \mathbb{R} \) above isn’t very precise, and thus isn’t a very good definition. The set of real numbers has a better definition, but it’s outside the scope of this course. For this semester we’ll make due with this intuitive notion of what a real number is.)

Numbers as subsets. \( \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \)

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Exercises

Decide whether the following statements are true or false.

1) $3 \in \{7, 4, -10, 17, 3, 9, 67\}$

2) $4 \in \{14, 44, 43, 24\}$

3) $\frac{1}{3} \in \mathbb{Z}$

4) $-5 \in \mathbb{N}$

5) $\frac{271}{113} \in \mathbb{Q}$

6) $-37 \in \mathbb{Z}$

7) $5 \in \mathbb{R} - \{4, 6\}$

8) $\{2, 4, 7\} \subseteq \{-3, 2, 5, 4, 7\}$

9) $\{2, 3, 5\} \subseteq \{2, 5\}$

10) $\{2, 5, 9\} \subseteq \{2, 4, 9\}$

11) $\{-15, \frac{3}{4}, \pi\} \subseteq \mathbb{R}$

12) $\{-15, \frac{3}{4}, \pi\} \subseteq \mathbb{Q}$

13) $\{-2, 3, 0\} \subseteq \mathbb{N}$

14) $\{-2, 3, 0\} \subseteq \mathbb{Z}$

15) $\{\sqrt{2}, 271\} \subseteq \mathbb{R}$

16) $\{\sqrt{2}, 271\} \subseteq \mathbb{Q}$