FRACTURE OF PRESSURIZED METAL AND GRAPHITE/EPOXY CYLINDERS FOR APPLICATIONS TO PRESSURE VESSEL DESIGN

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ABSTRACT

In this paper, a fracture criterion is discussed for the design against fracture in pressurized cylindrical vessels. The design criterion incorporates correction factors to account for the geometry of the cylinders, the thickness and the plastic deformation (à la Dugdale). The criterion may now be used to predict catastrophic failures in pressurized metal cylinders by knowing only material properties. Moreover, a correlation function is shown to exist between a plate and a cylinder. This correlation function may now be used to predict failures in cylindrical vessels from experimental data performed on flat plates of same material. Comparison of experimental data on pressurized cylinders made of metal as well as of graphite/epoxy is very good.

KEYWORDS

Bursting pressure; fracture; pressure vessels; failure of composite vessels; fracture criterion.

INTRODUCTION

The list of natural shell-like structures is long, and the strength properties of some of them are remarkable. It is logical, therefore, for engineers to utilize them in man-made structures. In doing so, however, one must understand the fundamental laws that govern the strength of such structures for they are not immune to failures.

Let us examine the concept of a pressure vessel a little more closely. Consider for example a balloon that is pressurized. Let us next use a pin to puncture a hole in the balloon. Experience tells us that two things may happen. If the hole is small, the balloon will simply lose pressure. If the hole is large, the balloon will explode. One conjectures, therefore, that for a given pressure there exists a critical flaw size beyond which catastrophic failure may take place. And vise versa, for a given flaw size, there exists a critical pressure beyond which the balloon will explode.

Pressure vessels do resemble balloons and like balloons are subject to puncture and explosive loss. Thus one may assume the existence of a relationship between the inherent variables of the system, i.e.

$$q_{crit} = f(\text{vessel geometry, material properties, flaw shape and size, etc.}) \quad (1)$$

such a relationship, we refer to as a Failure criterion or Fracture criterion. For its derivation, two ingredients are necessary: (i) the stress field due to the presence of a discontinuity, e.g. a crack and (ii) an energy balance for crack initiation.

FRACTURE CRITERION FOR METALS

The principal task of Fracture Mechanics is precisely the prediction of such failures in the presence of sharp discontinuities. An extensive study of the subject by the author (Folias, 1965, 1967) has led to the derivation of a fracture criterion (Folias, 1974) incorporating a geometry and a plasticity correction factor. The criterion may now be used to predict catastrophic failures in pressurized metal cylinders knowing only material properties. Moreover, in view of some recent developments (Folias, 1988) it is now possible to incorporate a correction factor for the effect of thickness[1] provided that the pressure vessel remains to be shallow (a pressure vessel is said to be shallow if $L/R \leq 0.1$, where L represents a linear dimension). Without going into the mathematical details, the fracture criterion reads

(i) for cylindrical vessels with an axial crack[2]:

$$(q_0 R/h) \sqrt{1 + 0.317\,\lambda^2}\, f_{thickness} = F \quad (2)$$

(ii) for cylindrical vessels with a peripheral crack:

$$(q_0 R/2h) \sqrt{1 + 0.05\,\lambda^2}\, f_{thickness} = F \quad (3)$$

where

$$F \equiv \frac{2\sigma^*}{\pi} \cos^{-1} \{\exp[-\frac{\pi K^2}{8\sigma^{*2} c}]\} \quad (4)$$

$$f_{thickness} \approx 1 + 0.07\, e^{-0.08\,(c/h)^2} \text{ for; } \quad (5)$$

$$0.5 \leq c/h < \infty \text{ and } \nu = 0.33$$

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[1] As related to plane stress or plane strain.
[2] It may be noted that the correction factor due to the geometry is actually a a very complicated function which may be approximated within a 6% error by the simple expression under the square root. For a more accurate result, see (Folias, 1969a).

and

q_0 = uniform internal pressure

R = radius of the vessel

h = wall thickness

K = fracture toughness

$\sigma^* = (3/4)\sigma_Y + (1/4)\sigma_u$

$\lambda^2 \equiv \{12(1-\nu^2)\}^{1/2} \dfrac{c^2}{\sqrt{Rh}}$

2c = the crack length (through the thickness)

σ_Y = material yield stress

σ_u = material ultimate stress

ν = Poisson's ratio.

It should be emphasized that the above equations (2) and (3) are based on the following assumptions:

A1: shallow shell theory
A2: homogeneous and isotropic material
A3: plasticity correction à la Dugdale
A4: no bulging effects present.

The designer may also notice that cracked pressure vessels in general exhibit a reduced resistance to fracture initiation than the corresponding case of plates. This result is a consequence of the interaction which exists, at the vicinity of the crack, between bending and stretching due to the curvature of the vessel. In the event that crack length is relatively small, i.e.

$$c < 0.08 \left(\frac{K}{\sigma^*}\right)^2 \quad (6)$$

then the function on the right-hand side of equation (4) becomes

$$F \approx \sigma^* \quad (7)$$

and equations (2) and (3) reduce considerably. For example equation (2) now reads:

$$q \approx \frac{1}{\sqrt{1 + 0.317\,\lambda^2}} \; \frac{1}{f_{\text{thickness}}} \left(\frac{2\sigma^* h}{R}\right). \\ \approx \frac{0.93}{\sqrt{1 + 0.317\,\lambda^2}} \left(\frac{2\sigma^* h}{R}\right) . \quad (8)$$

The above equation may now be used to predict bursting pressures in a cylindrical vessel by assuming a nominal statistical crack length typical for the material used. Moreover, for small temperature variations, the designer may choose appropriate values for σ^* and K. The reader should be cautioned that the correction factor for the effect of thickness is based on the assumption that the pressure vessel remains shallow. For the range of $0.5 \leq c/h < \infty$, this reflects a stress variation of approximately 7%. While this variation appears to be almost negligible for bursting pressures, it does however have a pronounced effect on the fatigue life of the vessel. In fact, it may very well reflect a reduction in the fatigue life of the structure of as much as 30% or 40%.

COMPARISON BETWEEN THEORY AND EXPERIMENTS

In judging the adequacy of a theory, one often compares theoretical and experimental results. Therefore, in the following we compare our results with some of the experimental data existing in the literature. In Figs. 1 and 2, we compare the theoretical results (Anderson and Sullivan, 1965) carried out on 6 diameter, 0.060 in. thick cylinders with through axial cracks. The material was 2014-T6Al and the cylinders were tested at -423°F and at room temperature. The agreement is good.

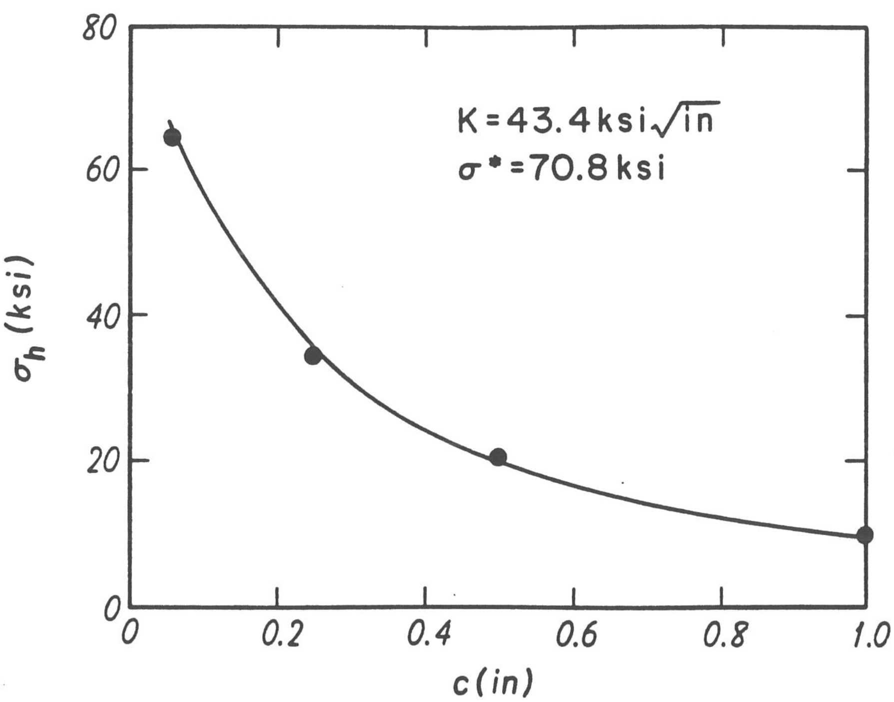

Fig. 1. Comparison between theory and experiment for 2014-T6al cylindrical vessels at room temperature.

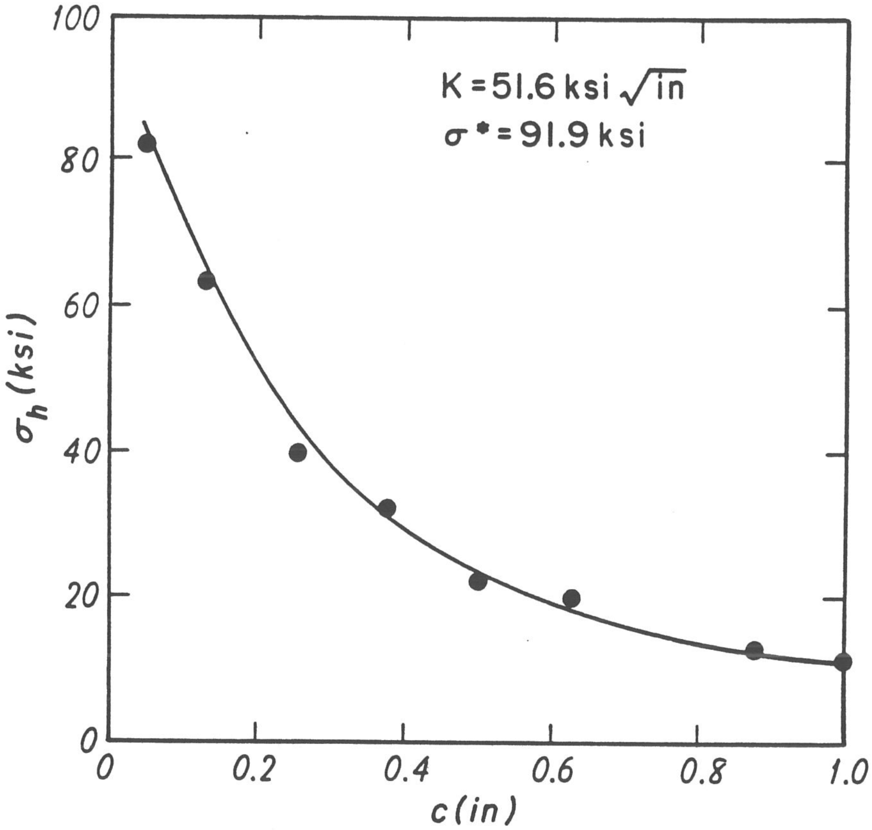

Fig. 2. Comparison between theory and experiment for 2014-T6Al cylindrical vessels at -423°F.

In Fig. 3 we compare the theoretical and experimental results (Duffy, 1965) on 30 in. diameter 3/8 in. thick pipes with through thickness cracks. The material tested was x-52 plain carbon (semi-killed). The agreement is good. Comparisons with other experimental data shows equally good agreement. This close agreement between the theoretically predicted fracture strengths and the experimental data suggests that equations (2) and (3) can be used to predict failure in pressurized vessels knowing crack length and material properties.

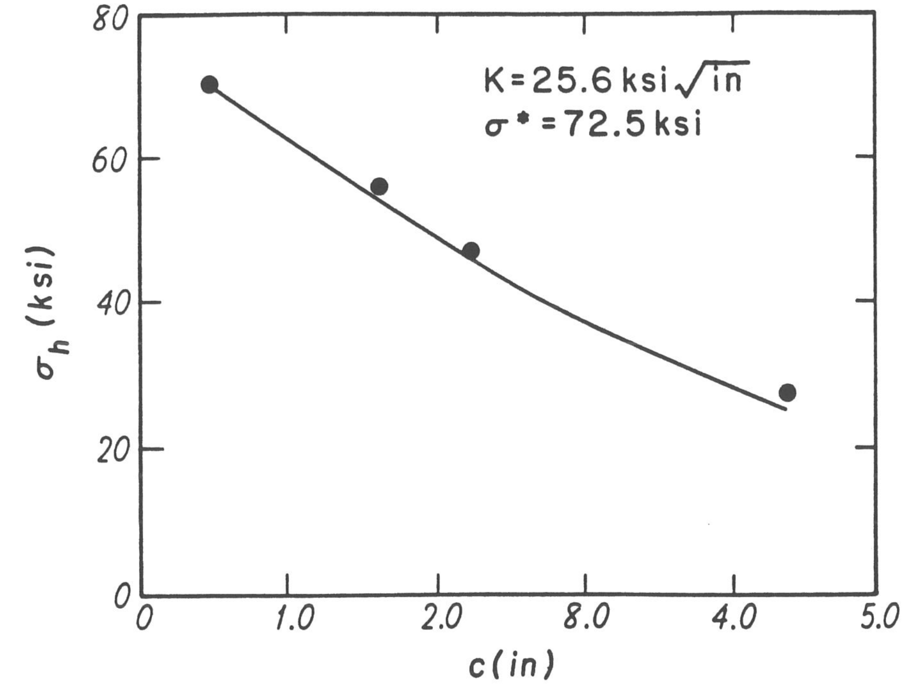

Fig. 3. Comparison between theory and experiment for x-52 plain carbon pipes.

CORRELATION BETWEEN PRESSURE VESSELS AND PLATES

As it was previously noted, in a cracked shell the inherent consequence of an initial curvature is the presence of an interaction between bending and stretching which results to higher stress levels at the crack, than those found in a similarly loaded flat plate. Moreover, it can be shown (Folias, 1965) that if one lets the radius of the shell structure R to tend to infinity, one recovers precisely the corresponding stress field in a similarly loaded flat plate. It is logical, therefore, to ask whether it is possible to correlate flat sheet behavior with that of initially curved specimens. In experimental work on brittle fracture, for example, considerable time could be saved if one could predict the response of pressurized vessels from tests on flat plates of the same material. Such a correlation function has been derived (Folias, 1969b) and it is of the form

$$\frac{\sigma_{hoop}}{\sigma_{plate}} = \frac{1}{\sqrt{1 + 0.317\,\lambda^2}} \,. \qquad (9)$$

It is also possible to obtain a relation between the critical crack length in a cylinder and the critical crack length in a similarly loaded cracked plate, in particular

$$\left[\frac{c_{cylinder}}{c_{plate}}\right] = \frac{1}{1 + 0.317\ \lambda^2} \quad (10)$$

A comparison with experimental data on metal pipes containing axial notches (Kihara et al, 1966), shows a fairly good agreement (see Fig. 4). The reader should note that the experimental data are based on notches rather than cracks and, naturally, reflect a higher value for hoop stress.

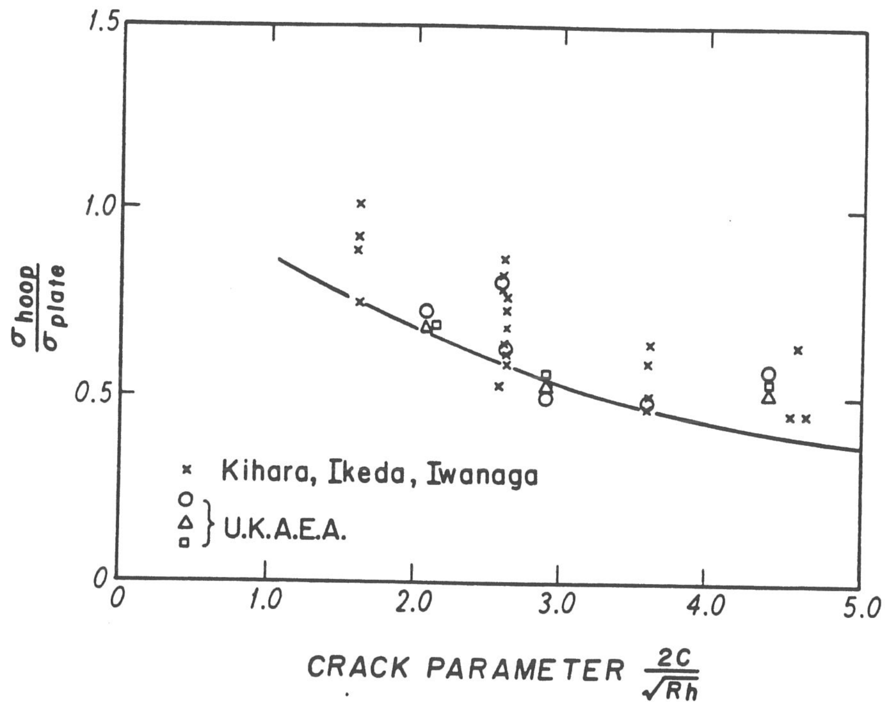

Fig. 4. Correlation between fracture stress ratio of pipe and flat plate vs. $(2c/\sqrt{Rh})$.

DAMAGE OF COMPOSITE CYLINDERS

Although the correlation function (eq. 9) was derived on the basis of a homogeneous and isotropic material, the author believes that it reflects the dominant term of a geometrical inherent property that exists between shell structures and similarly loaded flat plates of the same material.

Needless to say that such a property is of great practical significance to a designer for, it is now possible to predict catastrophic failures in shell-like structures from experimental data accumulated in the laboratory on flat plates. This is an advantage that has not only important economic implications but also the experiments that need to be carried out are much simpler.

In order for us to verify this hypothesis, we will apply it to the case of a composite material. Unlike the fracture of metals, which is well characterized by the principles of linear elastic fracture mechanics, the fracture of composite structures involves a complex interaction of fiber breaks, matrix cracks and interply delaminations.

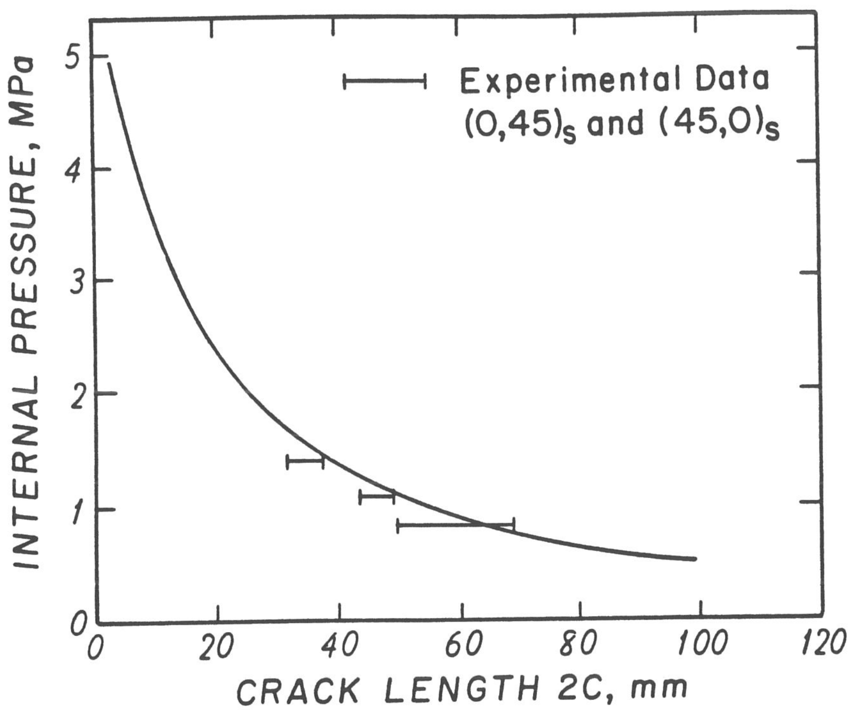

Fig. 5. Comparison between theory and experiment for graphite/epoxy cylinders with axial slits.

Without going into the details, in Fig. 5 we compare the results of eq. (9) with experimental data available in the literature (Graves and Lagace, 1985) on pressurized graphite/epoxy cylinders. The cylinders were 610 mm long and 305 mm in diameter and were fabricated from Hercules A370-5H/3501-6 prepreg fabric in quasi-isotropic four-ply configurations: $(0,45)_s$ and $(45,0)_s$. The cylinders were slit in the longitudinal direction and were pressurized[3]. The agreement is fairly good. It may be noted that if one uses the exact geometry correction factor (see footnote 2) the theoretical curve shifts a little lower and the agreement improves slightly.

CONCLUSIONS

A correlation function has been presented by which the failure of pressurized metal, as well as graphite/epoxy, cylinders with cracks can be predicted from test data on flat plates. Although in the case of metals this correlation function is well established, further experimental evidence is required in composite structures in order to establish its validity and its potential use.

REFERENCES

Anderson, R. B., and Sullivan, T. L. (1965). NASA TN D-3252.
Duffy, R. (1965). Symposium on Line Pipe Research, Dallas, Nov. 17-18.
Folias, E. S. (1965). Intern. J. Fracture Mech., 1, 20-46.
Folias, E. S. (1967). Intern. J. Fracture Mech, 3, 1-11.
Folias, E. S. (1974). Prospects of Fracture Mechanics, Noordhoff International, 405-418.
Folias, E. S. (1988). ASTM STP 969, in press.
Folias, E. S. (1969a). Intern. J. Fracture Mech., Vol. 5, No. 4, 327-346.
Folias, E. S. (1969b). UTEC 69-063, University of Utah, 1-74.
Kihara, H., Ikeda, K., and Iwanaga (1966). Document No. x-371-66, International Institute of Welding, Delft, Holland.
Graves, M. J., and Lagace, P. A. (1985). Composite Structures 4, 75-91.

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[3]In composites, it is the size of the flaw and not its shape that governs the fracture stress. Specimens with holes or slits of the same size failed at the same stress.