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Estimating plastic zone sizes

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Inasmuch as the Dugdale model [1] is an approximate model for a pseudo elasto-plastic analysis, the following simple method for estimating plastic zone sizes ahead of the crack tip is somewhat justifiable.

1. Plastic zone size for a plate

Consider the tensile stress σ as acting on a thin plate containing a crack of length $2a_p$ (see

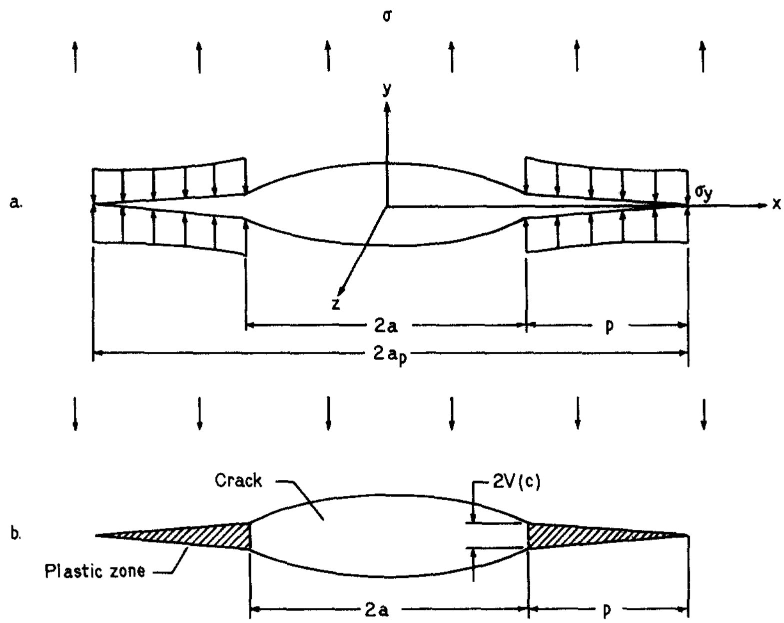

Figure 1. (a) Internal stress distribution used in the Dugdale model of elastic-plastic deformation near a crack of length $2a$ under plane-stress tensile loading. (b) Displacements $2V$ associated with crack opening.

figure 1). Then the singular term of the stress $\sigma_y^{(1)}$ along the crack prolongation at the point $x=a_p+r$ is

$$\sigma_y^{(1)} = \sigma \left(\frac{a_p}{2r}\right)^{\frac{1}{2}} + \ldots \tag{1}$$

Consider next the yield compressive stress as acting on a thin plate containing a crack of length $2p$. Similarly, the singular term of the stress $\sigma_y^{(2)}$ at the same point $x=a_p+r$ is

$$\sigma_y^{(2)} = -\sigma_Y \left(\frac{p}{2r}\right)^{\frac{1}{2}} + \ldots \,. \tag{2}$$

Superimposing, therefore, the two solutions and requiring that the stress σ_y be finite, one has

$$\sigma \left(\frac{a_p}{2r}\right)^{\frac{1}{2}} - \sigma_Y \left(\frac{p}{2r}\right)^{\frac{1}{2}} = 0 \tag{3}$$

or

$$\alpha \equiv \frac{a}{a_p} = 1 - \left(\frac{\sigma}{\sigma_Y}\right)^2 . \tag{4}$$

In Figure 2 experimental values are shown in relation to the curves calculated from Dugdale's equation and equation (4).

2. Plastic zone size for a cylindrical shell

Proceeding along the same lines as in the derivation of equation (4) one, in view of reference [2], has

$$\sigma_h M(a_p) \left(\frac{a_p}{2r}\right)^{\frac{1}{2}} - \sigma_Y M(p) \left(\frac{p}{2r}\right)^{\frac{1}{2}} = 0 \tag{5}$$

where the function $M(a)$ is a geometry correction factor and for an axial crack it may be approximated (within a 6 percent error) by

$$M(a) \simeq (1 + 0.34\lambda^2)^{\frac{1}{2}} . \tag{6}$$

Next, solving for the ratio a/a_p one has

$$\alpha \equiv \frac{a}{a_p} = 1 - \left(\frac{\sigma_h}{\sigma_Y}\right)^2 \frac{1 + 0.34\lambda^2 \left(\frac{1}{\alpha}\right)^2}{1 + 0.34\lambda^2 \left(\frac{1}{\alpha} - 1\right)^2} \tag{7}$$

It is interesting to note that the results obtained from equation (7) compare well with those of Figure 2 of reference [3].

3. Crack opening displacement

It is clear that as the plastic zones spread from the tip of the crack, the crack opening displace-

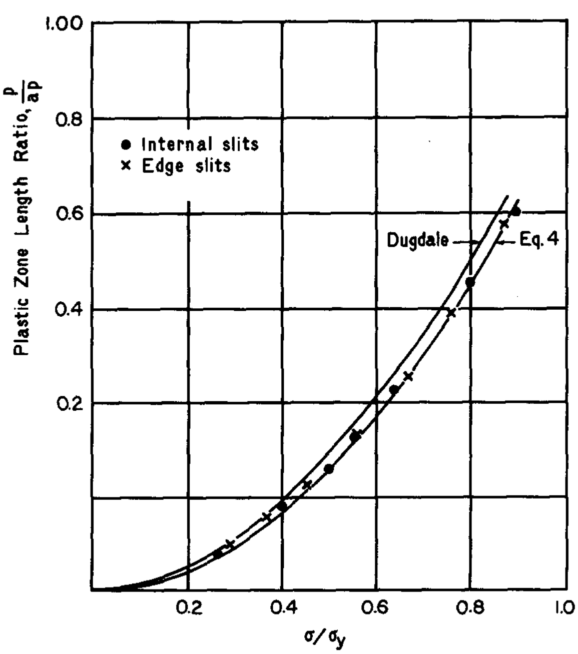

Figure 2. Comparison of plastic zone lengths.

ments $2V(a)$ produced at the tip will increase. These displacements, in the case of a plate, are related to the plastic zone size p by

$$2V(a) = \frac{8\sigma_Y a}{\pi E} \ln\left(\frac{a_p}{a}\right). \tag{8}$$

One may conjecture, therefore, that the crack opening displacement for a cylindrical shell will be of the same form except of an appropriate geometry correction factor, in particular

$$2V(a) = \frac{8\sigma_Y a}{\pi E} M(a_p) \ln\left(\frac{a_p}{a}\right). \tag{9}$$

Comparing equation (9) with Figure 9 of reference [3], one finds a fairly good agreement.

REFERENCES

[1] D. S. Dugdale, *Journal of Mechanics of Physics of Solids*, 8 (1960) 100.
[2] E. S. Folias, On the Effect of Initial Curvature on Cracked Flat Sheets, *International Journal of Fracture Mechanics*, 5, 4 (1969).
[3] F. Erdogan and M. Ratwani, Plasticity and the Crack Opening Displacement in Shells, *International Journal of Fracture Mechanics*, (1972).

RÉSUMÉ
Dans la mesure où le modèle de Dugdale est un modèle approximatif pour l'analyse pseudo élastoplastique, l'auteur propose une méthode simple pour estimer les dimensions de la zone plastique se trouvant en avant de l'extrémité d'une fissure.

ZUSAMMENFASSUNG
Da das Dugdale Modell ein annäherndes Modell för pseudo elastisch plastische Analyse ist, kann man das folgende Verfahren zur Bewertung der Abmessungen der plastischen Zonen an der Spitze eines Risses ziemlich rechtfertigen.