MATH 1170
MATHEMATICS FOR LIFE SCIENTISTS
Midterm 3, November 20, 2002

Do the following problems. Each problem is worth the number of points indicated. Write readable answers on the test but feel free to use and hand in extra paper. You may use your notes, books, and calculators.

1) Consider a population $N_i$ that obeys the dynamical system

$$N_{t+1} = \frac{4N_t}{1 + N_t} - h \cdot N_t$$

where the second term on the right side represents harvesting.

a) (7 pts) When $h = 0$, find the equilibria and determine their stability.

b) (9 pts) Find the equilibria as a function of the harvesting effort $h$. For which values of $h$ is there a positive equilibrium? For which values of $h$ is the positive equilibrium stable?

c) (9 pts) Determine the value of $h$ that gives the largest sustainable harvest. What is the harvest for this value of $h$?
2) For each of the following tell whether the statement must be true. If your answer is yes, cite a theorem that supports your answer. If your answer is no, give an example that supports your answer.

a) (5 pts) If a function $f(x)$ satisfies $f(0) = -1$ and $f(2) = 1$, then there is a $\bar{x}$ between 0 and 2 for which $f(\bar{x}) = 0$.

b) (5 pts) A function $g(x)$ continuous for $1 < x \leq 2$ must have a global minimum.

c) (5 pts) If a function $f(x)$ is differentiable for $0 \leq x \leq 2$, $f(0) = 4$, and $f(2) = 8$, then there is a point $\bar{x}$ between 0 and 2 for which $f'(\bar{x}) = 2$.

d) (5 pts) If the function $h(x)$ is differentiable, $h(0) = 0$, $h(1) = 0$, and $h(2) = 0$, then the function $h(x)$ must have at least two critical points.

e) (5 pts) The functions $f(x) = \ln(x)$ and $g(x) = x/(x + 1)$ intersect for some $x > 0$. 
3) a) (9 pts) Give the formula for a function \( f(x) \) whose leading behavior at 0 is \( x \) and whose leading behavior at infinity is \( 1/x^2 \). Sketch the function.

b) Compute the limits:
   i) (8 pts) \( \lim_{x \to 0} \frac{x^2}{\sin(x)} \)

   ii) (8 pts) \( \lim_{x \to \infty} \frac{\sin(x)}{x^2} \)
4) (25 pts) The graph of the function \( f(x) = x^2 - 4 \) is given below. Starting with initial guess \( x_0 = 3 \), show graphically the value Newton’s method would give as the next approximate solution, \( x_1 \), to the equation \( f(x) = 0 \). Algebraically find \( x_1 \) and \( x_2 \) using Newton’s method.