MATH 1170
MATHEMATICS FOR LIFE SCIENTISTS
Computer Assignment IX
Due October 29, 2001

PROBLEM

We will study the stability of the equilibria of Ricker discrete-time dynamical system

\[ x_{t+1} = \lambda x_t e^{-x_t}. \]

This system is much like the logistic dynamical system studied in class. The value \( x_t \) represents the number of fish (measured in millions). The parameter \( \lambda \) represents the number of new fish produced per fish in the absence of competition. In this case, the per capita reproduction is an exponentially decreasing function of the number of fish.

You will need to type \texttt{ireda(iter)} to get \texttt{iterplot}, \texttt{cobweb} and \texttt{iterplot2} to work.

a. Input the updating function as a function and graph it for a couple of values of \( \lambda \). Mark the equilibria.

b. Solve for the equilibria as a function of \( \lambda \) (there should be two, and \texttt{solve} should find them both).

c. One of your equilibria is negative when \( \lambda \) is too small. What is the critical value of \( \lambda \)? Does this make sense?

d. Find the derivative of the updating function at each equilibrium (as a function of \( \lambda \)).

e. For what values of \( \lambda \) is the positive equilibrium stable?

f. For one value of \( \lambda \) where the positive equilibrium is stable with a \textbf{positive} derivative, use the \texttt{cobweb} command starting from a reasonable initial condition.

g. For one value of \( \lambda \) where the positive equilibrium is stable with a \textbf{negative} derivative, \texttt{cobweb} starting from a reasonable initial condition.

h. Find the tangent line to the positive equilibrium in this case.

i. Use \texttt{iterplot2} to compare the dynamics with the original function and the dynamics with the tangent line (which you’ll have to input as a function) starting from an initial condition that is relatively near the equilibrium. Are the solutions similar?

j. For one value of \( \lambda \) where the positive equilibrium is unstable, \texttt{cobweb} starting from a reasonable initial condition.

k. The case with \( \lambda = 17 \) (along with lots of other cases) is “chaotic”. One of the properties of chaotic systems is “sensitivity to initial conditions”. Use \texttt{iterplot2} to start the system from two initial conditions which are very close (say differing by 1 fish – recall that \( x \) is measured in millions of fish) and describe what happens. Make sure to run it long enough to see something interesting. If you don’t find this interesting, try the same thing with a value of \( \lambda \) that produces a stable equilibrium.