Math 5610/6860
Homework #5, due Tue Nov 8

Note: Please see the sample code for Prob 1 in class website.

1. B&F 3.5.4(d) and 3.5.6(d). Produce a plot of the points and their natural cubic spline interpolant (as in the sample code). The easiest way is to proceed as in Example 2 p150, i.e. form the system of Theorem 3.11, solve it for the coefficients of the spline in each subinterval and then evaluate. You are not allowed to use Matlab’s spline or interp1 for this problem.

2. Assume every time we compute a function $f(x)$ we compute in fact $f_\epsilon(x)$, such that $|f(x) - f_\epsilon(x)| \leq \epsilon$. This systematic error could come from measurement errors or simply from using floating point to represent $f(x)$. The error made with the forward difference approximation to the derivative using $f_\epsilon$ instead of $f$ is

$$\left| \frac{f_\epsilon(x+h) - f_\epsilon(x)}{h} - f'(x) \right| \leq 2\epsilon \frac{1}{h} + \frac{h}{2} M,$$

for some $\xi$ between $x$ and $x+h$ and where $M$ is such that $|f''(\xi)| \leq M$ for $\xi \in [x, x+h]$.

(a) What is the step size $h_*$ that minimizes this upper bound?
(b) Let $f(x) = e^x$. Plot in log log scale (loglog in Matlab) the approximation error for $f'(1)$ using the forward difference formula for $h = 10.~^{\circ}(0:-1:-20)$
(c) Explain your results. Is the best step size $h$ in your graph consistent with the $h_*$ you derived?

3. (K&C 7.2.12) Derive a formula for approximating $\int_1^3 f(x)dx$ in terms of $f(0)$, $f(2)$ and $f(4)$. Your formula should be exact for all polynomials of degree $\leq 2$. Note: The node $x = 4$ is indeed outside of the integration interval. Use the undetermined coefficients method.

4. B&F 4.3.16. To find the degree of precision you need to find the largest $k$ such that the formula is exact for the monomials $x^0, x^1, \ldots, x^k$.

5. B&F 4.4.8 a,b,c. (Comparison of composite trapezoidal, midpoint and Simpson’s quadratures)

6. B&F 4.7: 1 through 4 for functions (a) and (b). (Gaussian quadrature)