Show all work. Write your answer in the space provided. Please box your answer. You may use one 3x5 note card, but it must be handed in with your test. You may use a calculator. Your graphs must be detailed for full credit.
Math 1220-1   Exam 3

1. Write the fourth degree Taylor Polynomial for \( g(x) = (1 - x)^{3/5} \) for \( a = 2 \).

\[
\begin{align*}
g(x) &= (1 - x)^{3/5} \\
g'(x) &= -\frac{3}{5} (1 - x)^{-2/5} \\
g''(x) &= \frac{6}{5^2} (1 - x)^{-7/5} \\
g'''(x) &= -\frac{42}{5^3} (1 - x)^{-13/5} \\
g^{(4)}(x) &= \frac{48(13)}{5^4} (1 - x)^{-18/5}
\end{align*}
\]

\[
\begin{align*}
P_4(x) &= 1 + \frac{3}{5} (x - 2) - \frac{6}{5^3} (x - 2)^2 + \frac{48}{5^4} (x - 2)^3 - \frac{48(13)}{5^4} (x - 2)^4
\end{align*}
\]

2. Use Simpson’s Rule with \( n = 6 \) to approximate \( \int_1^4 \frac{\sin x}{x} \, dx \).

\[
h = \frac{4 - 1}{6} = \frac{1}{2}
\]

\[
\int_1^4 \frac{\sin x}{x} \, dx \approx \frac{\frac{1}{3}}{3} \left[ \frac{\sin 1}{1} + 4 \frac{\sin 1.5}{1.5} + 2 \frac{\sin 2}{2} + 4 \frac{\sin 2.5}{2.5} + 2 \frac{\sin 3}{3} + 4 \frac{\sin 3.5}{3.5} + \frac{\sin 4}{4} \right]
\]

\[
= 0.8120
\]
3. Approximate the real root of \( y = 3x^5 - x^3 + 6x - 1 \) to four decimal places using Newton's method.

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

\[ x_1 = 0 \]

\[ x_2 = 0.1666667 \ldots \]

\[ x_3 = 0.167382 \ldots \]

\[ x_4 = 0.167382 \ldots \]

Root @ \( x = 0.16738 \)

4. A space probe uses the gravitational pull of a massive object in space to sling shot the probe in the desired direction, and let's say this sling shot trajectory follows a parabolic curve with the massive object at the focus of the parabola. What is the equation of a probe's path if at \( t = 0 \) minutes, the probe is at the origin, and at \( t = 3 \) minutes, the probe is 5000 miles away from the object?

\[ |QL| = |QF| = 5000 \]

\[ \sqrt{(3+p)^2 + (y-y)^2} = 5000 \]

\[ 3 + p = 5000 \]

\[ p = 4997 \]

\[ y^2 = 4(4997)x \]
5. Display with a picture the optical property of a hyperbola.

6. Satchel the dog is chasing Bucky the cat around the yard. Satchel isn’t the smartest puppy from the litter, and Bucky likes toying with Satchel, so Bucky’s path follows the equation \( r^2 - 5r \cos \theta + 7r \sin \theta = 0 \). Bucky manages to stay ahead of Satchel for two hours, and then Bucky runs up a tree. Find the Cartesian equation of the path Bucky follows. **Extra Credit** By completing the square, identify the conic which is now etched in the lawn as a result of the chase. (Circle: \((x - c)^2 + (y - d)^2 = r^2\); Ellipse: \(\frac{(x-a)^2}{a^2} + \frac{(y-b)^2}{b^2} = 1\); Hyperbola: \(\frac{(x-a)^2}{a^2} - \frac{(y-b)^2}{b^2} = 1\).)

\[
\begin{align*}
\gamma^2 - 5r \cos \theta + 7r \sin \theta & = 0 \\
\chi^2 + y^2 - 5x + 7y & = 0 \\
\chi^2 - 5x + (\frac{5}{2})^2 + y^2 + 7y + (\frac{7}{2})^2 & = (\frac{5}{2})^2 + (\frac{7}{2})^2 \\
\left(\chi - \frac{5}{2}\right)^2 + \left(y + \frac{7}{2}\right)^2 & = \frac{25 + 49}{4}
\end{align*}
\]
7. The figure below is the graph of the circle \( r = 2 \) and the circle \( r = 4 \cos \theta \).

(a) Find all the intersection points of the two graphs.

\[
\begin{align*}
2 &= 4 \cos \theta \\
\frac{1}{2} &= \cos \theta \\
\theta &= \frac{\pi}{3} + 2\pi n
\end{align*}
\]

**Points of Intersection:** \((2, \frac{\pi}{3}), (2, -\frac{\pi}{3})\)

(b) Set up, but do not solve, the integral needed to find the area of the shaded region.

\[
A = \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2^2 - (4 \cos \theta)^2 \ d\theta
\]