Class #33
Most popular response to

- What did the students want to prove?
  - The angle bisectors of a square meet at a point.

- A *square* is a convex quadrilateral in which all sides are congruent and all angles are right angles.
Claim: The angle bisectors of a square meet at a point.

- Proof: Consider the angle bisector of $\angle BAC$.

This is too complicated. Let us try a different approach.
Claim: The angle bisectors of a square meet at a point.

Proof: Let \( a \) be the angle bisector of \( \angle A \), \( b \) the angle bisector of \( \angle B \), etc. Consider the triangles \( \triangle ACB \) and \( \triangle ACD \).

Since \( AB \cong AD \), \( BC \cong CD \), by SSS, we have that triangles \( \triangle ACB \) and \( \triangle ACD \) are congruent.

By definition of congruent triangles \( \angle BAC \cong \angle DAC \). This and the fact that \( C \) is in the interior of the angle \( \angle BAD \) (consequence of the definition of convex quadrilateral), imply that the ray \( \overrightarrow{AC} \) is the angle bisector of \( \angle BAD \), that is \( a = \overrightarrow{AC} \).

Similarly, we conclude that \( c = \overrightarrow{CA} \).

By Prop 3.1. \( a \cap c = \overrightarrow{AC} \).
Consideration of triangles ΔBDA and ΔBDC, would in the identical manner give us that $b \cap d = BD$.

We now have (using set theory)

$$a \cap c \cap b \cap d = AC \cap BD$$

The intersection of the angle bisectors is the intersection of the diagonals, and we have proved that the diagonals of a convex quadrilateral intersect in a point. Therefore, the angle bisectors of a square intersect in a point.
Most popular response to

- What did the students prove?
  - The angle bisectors of a square are the diagonals.

- Could we prove this?

- How would you rephrase it so that it is a meaningful statement?
Conjectures \( \text{e}12 \)

- **Group 1:**
  - Def: A parallelogram is a convex quadrilateral whose opposite sides are parallel.
  - If the diagonals of a parallelogram \( \square ABCD \) lie on the angle bisectors such that \( BD \subset \text{bisector}(\angle ABC), BD \subset \text{bisector}(\angle ADC), AC \subset \text{bisector}(\angle DCB), AC \subset \text{bisector}(\angle DAB), \) then all four sides DC, AB, BC and DA are congruent.

- **Group 2:**
  - In a parallelogram the lines defined by opposite angle bisectors are either equal or parallel.

- **Group 3:**
  - 1: If all four sides are congruent, the angle bisectors of opposite angles are collinear, the bisectors of adjacent angles intersect at a point and are perpendicular.
  - 2: If opposite sides are parallel, then the angle bisectors of adjacent angles are perpendicular.
  - 3: If all 4 sides are different lengths, you are screwed.

- **Group 4:**
  - Bisectors of adjacent angles always meet. Therefore, one angle bisector will intersect at least 2 other angle bisectors and sometimes all 3.

- **Group 5:**
  - If all sides of a quadrilateral are congruent, then the intersection of all 4 angle bisectors is one point.
Def: A parallelogram is a convex quadrilateral whose opposite sides are parallel.

If the diagonals of a parallelogram $\square ABCD$ lie on the angle bisectors such that $BD \subset \text{bisector}(\angle ABC)$, $BD \subset \text{bisector}(\angle ADC)$, $AC \subset \text{bisector}(\angle DCB)$, $AC \subset \text{bisector}(\angle DAB)$, then all four sides $DC$, $AB$, $BC$ and $DA$ are congruent.
In a parallelogram the lines defined by opposite angle bisectors are either equal or parallel.
Group 3:

- 1: If all four sides are congruent, the angle bisectors of opposite angles are collinear, the bisectors of adjacent angles intersect at a point and are perpendicular.
  - Angle bisectors are collinear?

- 2: If opposite sides are parallel, then the angle bisectors of adjacent angles are perpendicular.

- 3: If all 4 sides are different lengths, you are screwed.
Group 4:

- Bisectors of adjacent angles always meet. Therefore, one angle bisector will intersect at least 2 other angle bisectors and sometimes all 3.
Group 5

- If all sides of a quadrilateral are congruent, then the intersection of all 4 angle bisectors is one point.

- Proof:
Conjectures @1

- **Group 1:**
  - Given a convex quadrilateral □ABCD, if the intersection of the angle bisectors emanating from any two opposite vertices is a segment, then those segments are diagonals and opposite angles are congruent.

- **Group 2:**
  - If all sides of a convex quadrilateral are congruent the angle bisectors meet at a unique point in the interior of the quadrilateral.

- **Group 3:**
  - The angle bisector of a convex quadrilateral intersects one side of a quadrilateral not containing the vertex it originated from. If it intersects both sides, then it contains its opposite vertex.

- **Group 4:**
  - Given a convex quadrilateral, if the intersection of the angle bisectors of the angles formed by the opposite vertices are equal to the diagonals, then the quadrilateral is a square.
  - A square is a quadrilateral with all four sides congruent and all four angles right angles.

- **Group 5:**
  - If a rectangle is not a square, then the angle bisectors intersect to form a square.
  - Rectangle - quadrilateral with four right angles and opposite sides congruent.
  - Square – rectangle with all sides congruent
Given a convex quadrilateral □ABCD, if the intersection of the angle bisectors emanating from any two opposite vertices is a segment, then those segments are diagonals and opposite angles are congruent.

- $m\angle DCA = 76.49^\circ$
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- $m\angle BAC = 30.00^\circ$
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- $m\angle DAC = 30.00^\circ$
- $m\angle BCA = 76.49^\circ$
Group 2:

- If all sides of a convex quadrilateral are congruent the angle bisectors meet at a unique point in the interior of the quadrilateral.

- Proof:
Group 3:

- The angle bisector of a convex quadrilateral intersects one side of a quadrilateral not containing the vertex it originated from. If it intersects both sides, then it contains its opposite vertex.
Given a convex quadrilateral, if the intersection of the angle bisectors of the angles formed by the opposite vertices are equal to the diagonals, then the quadrilateral is a square.

$m \angle DAB = 60.00^\circ$

AD = 4.19 cm   AB = 4.19 cm
DC = 4.19 cm   BC = 4.19 cm
Group 5:

- If a rectangle is not a square, then the angle bisectors intersect to form a square.
- Rectangle - quadrilateral with four right angles and opposite sides congruent.
- Square – rectangle with all sides congruent