MATH1220: Midterm 3 Practice Problems

The following are practice problems for the second exam.

1. Compute the following limits:
   (a) \[ \lim_{x \to 1} \frac{x^2 - 2x + 1}{\sin(\pi x)} \]
   (b) \[ \lim_{x \to 0} \frac{e^x - e^{-x}}{2 \sin x} \]
   (c) \[ \lim_{x \to \infty} \frac{(\ln x)^2}{2x} \]
   (d) \[ \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \]

2. Evaluate the following integrals:
   (a) \[ \int_{-\infty}^{1} e^{4x} \, dx \]
   (b) \[ \int_{5}^{\infty} \frac{x}{1 + x^2} \, dx \]
   (c) \[ \int_{-\infty}^{\infty} \frac{dx}{(x^2 + 9)^2} \]
   (d) \[ \int_{0}^{3} \frac{dx}{x^2 - 2x - 3} \]
   (e) \[ \int_{-4}^{0} \frac{dx}{(x + 3)^2} \]

3. Write an explicit formula for the \(n\)-th term of the sequence. Then determine whether the sequence converges or diverges. If it converges, find what number it converges to:
   (a) \( a_1 = 7, \quad a_{n+1} = a_n \left(\frac{2}{3}\right) \)
   (b) \(-1, 2, 5, 8, 11, \ldots \)
   (c) \(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \ldots \)

4. Find the limit of the sequence \(a_n = \frac{2n^3}{5n^3 - 2n + 2} \).

5. Show that the sequence \(a_n = \frac{n}{n+1} \left(2 - \frac{1}{n^2}\right) \) converges using the monotone sequence theorem.

6. Determine the convergence/divergence of the following series:
   (a) \( \sum_{n=1}^{\infty} \frac{n}{n + 1} \)
7. Determine whether each of the following is absolutely convergent, conditionally convergent, or divergent:

(a) \[ \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}} \]

(b) \[ \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{e^k} \]

(c) \[ \sum_{n=1}^{\infty} (-1)^n \frac{n - 1}{n} \]