MATH1220: Midterm 1 Practice Problems

The following are practice problems for the first exam.

1. Compute the following derivatives:
   (a) \( D_x \left( \ln \sqrt{3x^2 + 2} \right) \)
   (b) \( D_x [e^{\sin x}] \)
   (c) \( \frac{d}{dx} \left[ \log_a (2x^2) \cos x \right] \)
   (d) \( \frac{d}{dx} \left[ 4^{3x^2+x+1} \right] \)

2. Compute the following integrals:
   (a) \( \int \frac{6x^2 + 16x}{x^3 + 4x^2 - 3} \, dx \)
   (b) \( \int_0^5 5xe^{x^2} \, dx \)
   (c) \( \int 3^x \, dx \)
   (d) \( \int \tan x \, dx \)

3. Find the inverse of the function \( f(x) = \frac{5x - 3}{2x - 1} \) and verify that it is actually the inverse by showing that \( f \circ f^{-1}(y) = y \) and \( f^{-1} \circ f(x) = x \).

4. Show that \( f(x) = x^5 + 2x^3 + 4x + \sin(\pi x) \) has an inverse (don’t try to find the inverse) and compute \( (f^{-1})'(7) \). (Hint: You can find an \( x \) such that \( f(x) = 7 \) by inspection)

5. Compute \( \frac{d}{dx} \left[ (1 + x^2)\cos x \right] \)

6. A radioactive substance loses 15% of its radioactivity in 2 days. What is its half-life?

7. Find the general solution to the following differential equation:
   \[ \frac{dy}{dx} + \frac{2y}{x+1} = (x+1)^3 \]

8. Use Euler’s Method with \( h = 0.5 \) to approximate the solution to
   \[ y' = 2y - 2x \quad \quad y(0) = 1 \]
   over the interval \([0, 1] \).
9. Sketch the solution to $y' = x^2 - y$, whose slope field is shown below, satisfying the initial conditions $y(-2) = 1$. 