MATH1220: Midterm 3 Study Guide

The following is an overview of the material that will be covered on the first exam.

§6.1 The Natural Logarithm Function
- The definition of the natural logarithm, including its derivative.
- Computing integrals of the form $\int \frac{du}{u}$.
- Using properties of logarithms to simplify the computation of derivatives (this is called logarithmic differentiation).
- Computing certain trig integrals (e.g., $\int \tan x \, dx$).

§6.2 Inverse Functions and Their Derivatives
- Finding the inverse of a function.
- Show a function has an inverse (without actually finding it). Our standard method for doing this is using Theorem A from §6.2.
- Checking that two functions are inverses of each other (we just show that $f \circ f^{-1}(y) = y$ and $f^{-1} \circ f(x) = x$).
- Using the Inverse Function Theorem.

§6.3 The Natural Exponential Function
- The definition of the natural exponential function, including its derivative.
- Computing derivatives of the form $D_x(e^u)$ and integrals of the form $\int e^u \, du$.

§6.4 General Exponential and Logarithmic Functions
- Derivatives and integrals involving general exponential functions (i.e., $a^x$ for arbitrary $a$) and general logarithms ($\log_a x$).
- Differentiating (or integrating) using the definition of $a^x$ (e.g., $D_x(x^x) = D_x(e^{x \ln x})$).

§6.5 Exponential Growth and Decay
- Solving word problems involving exponential growth/decay.
- Know that $\lim_{h \to 0} (1 + h)^{1/h} = e$.
- Solving separable differential equations by integration.

§6.6 First Order Linear Differential Equations
- Solving linear first-order differential equations using the integrating factor technique (I guarantee you will be asked to do this on the exam).
- Finding the general solution to such a differential equation.
- Finding a specific solution using given initial conditions.

§6.7 Approximations for Differential Equations
• Sketch a specific solution to a differential equation when given the slope field and an initial condition.
• Use Euler’s Method to approximate a solution to a differential equation.

§6.8 Inverse Trig Functions and Their Derivatives
• Deriving the identities from Theorem A (these are the ones that look like \( \sin(\cos^{-1} x) = \sqrt{1-x^2} \)).
• The derivatives of the six standard trig functions.
• Integrals involving inverse trig functions (e.g., \( \int \frac{3}{\sqrt{9-x^2}} \, dx \)).
• You will be given the derivatives of the inverse trig functions (see formula sheet).

§6.9 The Hyperbolic Functions and Their Inverses
• The definitions of the hyperbolic functions.
• The derivatives of the hyperbolic functions.
• Integrals involving inverse hyperbolic functions (e.g., \( \int \frac{dx}{\sqrt{x^2+1}} \)). There are multiple ways to do this integral. If you do a trig substitution (as in §7.4) you will get the algebraic expression for \( \sinh^{-1} x \).
• You will be given the derivatives of the inverse trig functions.

§7.1 Basic Integration Rules
• You should be able to integrate anything resembling 1-12, or 16,17 on p384 in the text.
• You will be given 13-15 on the formula sheet.
• You should be (very) comfortable with \( u \)-substitution.

§7.2 Integration By Parts
• Using integration by parts in definite and indefinite integrals.
• Recognizing when it is appropriate to try integration by parts.
• Repeated integration by parts.

§7.3 Some Trigonometric Integrals
• Integrals like \( \int \sin^n x \, dx \).
• Integrals like \( \int \sin^n x \cos^m x \, dx \).
• Integrals like \( \int \sin(mx) \cos(nx) \, dx \).
• Integrals like \( \int \tan^n x \, dx \).
• You will be given the half-angle formulas and the product identities.

§7.4 Rationalizing Substitutions
• Rationalizing substitutions for integrands involving \( \sqrt{ax+b} \).
• Trig substitutions for integrands involving \( \sqrt{a^2-x^2} \), \( \sqrt{x^2-a^2} \), or \( \sqrt{a^2+x^2} \).
§7.5 Partial Fraction Decompositions

- Integrating rational functions using partial fractions.
- Distinct or repeated linear factors.
- Distinct or repeated quadratic factors.
- The logistic differential equation will NOT be covered.

§7.6 Strategies for Integration

- Determining which technique(s) you should use to evaluate an integral.

§8.1 Indeterminate Forms of Type 0/0

- L’Hôpital’s Rule for forms of type 0/0
- Repeated L’Hôpital’s Rule for forms of type 0/0

§8.2 Other Indeterminate Forms

- L’Hôpital’s Rule for forms of type ∞/∞
- Indeterminate forms of type 0 · ∞ and ∞ − ∞
- Indeterminate forms of type 0^0, ∞^0, and 1^∞

§8.3 Improper Integrals: Infinite Limits of Integration

- Integrals of the form \( \int_a^\infty f(x) \, dx \)
- Integrals of the form \( \int_{-\infty}^b f(x) \, dx \)

§8.4 Improper Integrals: Infinite Integrands

- Integrals where the integrand is infinite at a limit of integration.
- Integrals of the form \( \int_a^b f(x) \, dx \) where the \( f(x) \) is infinite at some point in \((a, b)\).

§9.1 Infinite Sequences

- The definition of a sequence.
- Writing a general formula when given a list of terms or a recursive formula.
- Writing a recursive formula when given a general formula or a list of terms.
- Writing a list of the first few terms when given a general or recursive formula.
- Computing the limit of a sequence by computing the limit of a function (e.g., Example 3 in §9.1).
- Applying the Squeeze Theorem to show a sequence converges.
- Using the Monotone Sequence Theorem to show a sequence converges.

§9.2 Infinite Series

- Deriving a formula for the \( n \)-th partial sum of a series.
• Conditions under which a geometric series converges, and computing the sum of a geometric series.
• Finding the sum of a collapsing series.
• The $n$-th term test for divergence.
• The harmonic series.

§9.3 Positive Series: The Integral Test
• The Integral Test (make sure the hypotheses are satisfied).
• The $p$-series test.
• Using the integral test to bound the error on the $n$-th partial sum.

§9.4 Positive Series: Other Tests
• The Ordinary Comparison Test.
• The Limit Comparison Test.
• The Ratio Test.
• When each test is appropriate to try and what the hypotheses of the tests are.

§9.5 Alternating Series, Absolute Convergence, and Conditional Convergence
• The Alternating Series test (make sure the hypotheses are satisfied).
• The Absolute Convergence Test.
• The Absolute Ratio Test.
• Conditional Convergence.

§9.6 Power Series
• Finding the convergence set (or radius of convergence) of a power series in $x$ or $x - a$.

§9.7 Operations on Power Series
• Integrating and differentiating power series term by term.
• Adding and subtracting power series term by term.
• Multiplying and dividing power series.

§9.8 Taylor and Maclaurin Series
• Computing a Taylor series based at $x = a$.
• Taylor’s remainder formula.
• Note that a Maclaurin series is just a special case of a Taylor series (where $a = 0$).

§9.9 Taylor’s Approximation to a Function
• Approximating a function using the first few terms of the taylor polynomial based at $x = a$. 
• Finding an error for the remainder in such an approximation.

§10.5 The Polar Coordinate System

• Converting points and equations between polar and rectangular coordinates.
• Polar equations for lines, circles, and conics.

§10.6 Graphs of Polar Equations

• Limaçons, cardioids and spirals.

§10.7 Calculus in Polar Coordinates

• Computing areas via integrals in polar coordinates.