

Homework # 11

10.5 # 19: (3 pts, p. 732) Find a parametric representation for the surface: The plane that passes through the point (1,2,-3) and contains the vectors $\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$.

See Example 3 on page 729 for this. If $\mathbf{r}$ is a point on this plane, then that point can be represented as $\mathbf{r} = \mathbf{r}_0 + u\langle 1, 1, -1 \rangle + v\langle 1, -1, 1 \rangle$ where $u$ and $v$ are parameters used to identify a particular point on the plane. Expanding this equation gives:

\[
\mathbf{r} = \mathbf{r}_0 + u\langle 1, 1, -1 \rangle + v\langle 1, -1, 1 \rangle \\
= \langle 1, 2, -3 \rangle + u\langle 1, 1, -1 \rangle + v\langle 1, -1, 1 \rangle \\
= \langle 1 + u + v, 2 + u - v, -3 - u + v \rangle
\]

Parametric equations

\[
x = 1 + u + v \\
y = 2 + u - v \\
z = -3 - u + v
\]

11.1 # 12: (3 pts, p. 747) Two contour maps are shown. One is for function $f$ whose graph is a cone. The other is for a function $g$ whose graph is a paraboloid. Which is which and why?

The contour plot where the contours are spaced at equal intervals is the contour plot for the cone. This is because the slope (directional derivative going away from the $z$-axis) is constant for a cone. This same slope is not constant for a paraboloid. To show this better, here are contours and 3-dimensional plots of the paraboloid $z = 5 - 10x^2 - 10y^2$ and then the cone $z = 5 - 10\sqrt{x^2 + y^2}$. For the paraboloid:
For the cone:

11.2 # 8: (4 pts, p. 755) Find the limit, if it exists, or show that the limit does not exist.

\[
\lim_{(x,y) \to (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2}
\]

First find the limit as the point \((x, y) \to (0, 0)\) along the \(x\)-axis where \(y = 0\). For this limit:

\[
\lim_{(x,y) \to (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2} = \lim_{x \to 0} \frac{x^2}{2x^2} = \frac{1}{2}
\]

Now find the limit as the point \((x, y) \to (0, 0)\) along the \(y\)-axis where \(x = 0\). For this limit, use the fact that the limit of a product is equal to the product of the individual limits:

\[
\lim_{(x,y) \to (0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2} = \lim_{y \to 0} \frac{\sin^2 y}{y^2} = \left( \lim_{y \to 0} \frac{\sin y}{y} \right) \left( \lim_{y \to 0} \frac{\sin y}{y} \right) = 1
\]

Since different paths for \((x, y) \to (0, 0)\) lead to different values for the limit, the limit does not exist.