1. **True / False** Consider a system of linear equations with $n$ equations and $n$ unknowns, represented by the matrix equation $Ax = b$.

   (a) ________ The system can have at most $n$ pivots.
      **True:** Pivots by definition lie on the diagonal, and there are only $n$ diagonal entries.

   (b) ________ If the system has fewer than $n$ pivots, there is no solution.
      **False:** A system with fewer than $n$ pivots may in fact have infinitely many solutions, as for the system $x + 2y = 1$ and $-x - 2y = -1$.

   (c) ________ If $n = 4$, the following matrix might be used in the process of elimination:

   $\begin{bmatrix}
   0 & 0 & 0 & 1 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 1 & 0 \\
   1 & 0 & 0 & 0
   \end{bmatrix}$

      **True:** This is a *permutation matrix*, which corresponds to a row exchange. Row exchanges are used in elimination in some cases. (Careful of the terminology: this is not an “elimination matrix,” yet still may be used in elimination.)

2. Consider the system of linear equations with matrix form

   $\begin{bmatrix}
   -3 & 2 & 4 \\
   0 & 2 & -7 \\
   8 & -2 & 1 \\
   \end{bmatrix}
   \begin{bmatrix}
   x \\
   y \\
   z
   \end{bmatrix} =
   \begin{bmatrix}
   1 \\
   17 \\
   4
   \end{bmatrix}$.

   What is the first operation you would perform on this system using our elimination algorithm? Write the matrix $E$ so that left-multiplication by $E$ corresponds to this operation.
**Answer:** The first operation would be to eliminate the 8 in the (3,1) entry by adding $8/3$ times the first row to the third row. The corresponding matrix is

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{8}{3} & 0 & 1 \end{bmatrix}.$$ 

(A common incorrect answer was to exchange rows 2 and 3. In our algorithm, row exchanges are used only to create a nonzero pivot when there is a zero in a pivot position. The relevant pivot position at the beginning of the algorithm is the (1,1) position, which has the nonzero entry $-3$, so no row exchange is needed.)