Summary of Part 2 of the course

Solving systems of equations

An equation in $x, y$ cuts out a curve in the plane (e.g., straight line, parabola), a linear equation in $x, y, z$ cuts out a plane in 3-space. Solving systems of equations corresponds to finding points of intersection. §7.2 and p580.

There are two different approaches:

1. If one of the equations isn’t linear then you must use substitution §7.1.
2. If all $m$ equations in $n$ variables $x, y, z, ...$ are linear then solve using elementary row operations (ERO’s), i.e., §7.3,8.1.
   (a) write the augmented matrix.
   (b) use ERO’s to write the augmented matrix in echelon form.
   (c) rewrite the system of equations using the variables $x, y, z, ...$ and use back substitution.

There are three possibilities: (i) no solutions; (ii) unique solution; (iii) infinitely many solutions.

Matrices

Addition/subtraction/multiplication of matrices. Make sure you know the conditions on the dimension of the matrices for these operations to be well defined. §8.2.

We can only ‘divide’ by a matrix if it has an inverse. To find out whether an $n \times n$ matrix $A$ has an inverse (and to compute the inverse) you should §8.3

1. augment $A$ by the $n \times n$ identity matrix $I_n$.
2. use ERO’s to write the matrix $A|I_n$ in (row) reduced echelon form.
3. if the resulting matrix is of the form $I_n|A^{-1}$ then you’ve found the inverse! Otherwise $A$ has no inverse.

An application (but by no means the only reason for introducing inverses!) is to solve systems of $n$ linear equations in $n$ unknowns. Write the system as $AX = B$ and try to compute $A^{-1}$ as above. The system has a unique solution if and only if $A^{-1}$ exists and the solution is $X = A^{-1}B$.

Partial fractions

To write the partial fraction decomposition for a rational function: §2.7

1. if degree numerator $> \text{degree denominator}$ use long division giving a polynomial plus a rational function $f(x)/g(x)$ with degree $f < \text{degree } g$.
2. factor $g(x)$ into linear and quadratic terms (no complex numbers).
3. for every linear term \((Ax + B)\) include \(\frac{x}{Ax+B}\), if there is a quadratic term \((Cx^2 + Dx + E)\), include \(\frac{6x+c}{(Cx^2+Dx+E)}\). If there is a linear term squared \((Ax + B)^2\) include \(\frac{d}{(Ax+B)^2}\).....

4. add the terms from 3. to give something like, e.g.,

\[
\frac{(3a + b - c)x^2 + (4a - 3b + 2d)x + (a - b + c)}{g(x)}
\]

Set this equal to \(f(x)/g(x)\), giving you a system of equations

\[
\begin{align*}
3a + b - c &= \text{coefficient of } x^2 \text{ in } f \\
4a - 3b + 2c &= \text{coefficient of } x \text{ in } f \\
a - b + c &= \text{constant term from } f
\end{align*}
\]

Solve the system.

**Six steps to drawing graphs of rational functions**

Given a rational function \(r(x) = f(x)/g(x)\) with integer coefficients, draw \(y = r(x)\) as follows:

1. Is \(r(x)\) common function shifted/insert/inserted?
   \(\S 1.5.\)

2. Plot \(y\)-intercepts.

3. \(\text{(a) Plot } x\text{-intercepts, i.e., find zeros of } f(x) \text{ by factoring.}\)
   \(\text{(b) Plot vertical asymptotes, i.e., find zeros of } g(x).\)  \(\S 2.6.\)

4. Look at behaviour to far right and far left, i.e., find the horizontal / slant asymptotes:
   \(\S 2.6.\)
   \(\text{(a) if degree } f < \text{ degree } g \text{ then } y = 0 \text{ is asymptote.}\)
   \(\text{(b) if degree } f = \text{ degree } g \text{ then } y = a_n/b_n \text{ is asymptote, i.e., leading coeff of } f \text{ divided by leading coeff of } g.\)
   \(\text{(c) if degree } f > \text{ degree } g \text{ then use long division to write}\)

\[
r(x) = q(x) + \text{remainder}(x)/g(x)
\]

so that \(q(x)\) is a ‘slant’ asymptote.

5. Use cunning and plot at least one point between and one point beyond all zeros and vertical asymptotes.