Open book, calculators may only be used to check your solutions. Those with large writing may use the back of this page for working!

1. [8 points ] Shade the region representing the solutions of the system

\[
x \geq 0, \quad y \geq 0, \quad 3x + y \leq 15, \quad 4x + 3y \leq 30.
\]

To gain full points you have to record the vertices of the region on your graph.

**Solution:** Draw the lines \( x = 0, \ y = 0, \ 3x + y = 15 \) and \( 4x + 3y \leq 30 \) (1 point each). These four lines form the boundary of a quadrilateral (i.e., a region with four sides) which should be shaded (1 point). The vertices of the region should be marked on the graph. In addition to the origin \((0,0)\), the vertices are \((0,10), (5,0), \) and \((4,3)\) (1 point each; note that you do not get a point for marking the origin correctly!).

2. [2 points ] Find the maximum value of the function \( z = 4x + y \), and the point \((x, y)\) where the maximum occurs, subject to the constraints in part 1. above.

**Solution:** Substitute the four vertices into the function:

\[
(x, y) = (0, 0) \implies z = 0
\]
\[
(x, y) = (0, 10) \implies z = 10
\]
\[
(x, y) = (4, 3) \implies z = 19
\]
\[
(x, y) = (5, 0) \implies z = 20
\]

so the max value of \( z = 20 \) (1 point) occurs at \((5,0)\) (1 point).