9.5 16. Let \( R \) be the relation on the set of ordered pairs of positive integers such that \((a,b),(c,d)) \in R\) if and only if \( ad = bc \). Show that \( R \) is an equivalence relation.

**Proof:**

**Reflexive:** Let \( a, b \in \mathbb{Z}^+ \)

If \((a,b)R(a,b)\) then \( ab = ba \) \( \text{True} \)

So \( R \) is reflexive.

**Symmetric:** Let \( a, b, c, d \in \mathbb{Z}^+ \) such that \((a,b)R(c,d)\)

\( ad = bc \) \( \Rightarrow \) \( da = cb \) \( \Rightarrow \) \( cb = da \)

So \((c,d)R(a,b)\)

So \( R \) is symmetric.

**Transitive:** Let \( a, b, c, d, e, f \in \mathbb{Z}^+ \) such that \((a,b)R(c,d)\) and \((c,d)R(e,f)\)

\( ad = bc \) and \( cf = de \)

\[
\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a}{b} = \frac{e}{f}
\]

\( af = be \)

So \((a,b)R(e,f)\)

Therefore \( R \) is transitive.

(QED)
39. Suppose that \((S, \preceq_1)\) and \((T, \preceq_2)\) are posets. Show that \((S \times T, \preceq)\) is a poset where \((s, t) \preceq (u, v)\) if and only if \(s \preceq_1 u\) and \(t \preceq_2 v\).

\[ s, u, w \in S \text{ and } t, v \in T \]

**Proof:**

**Reflexive:** Let \(s \in S\) and \(t \in T\).

Since \(\preceq_1\) is reflexive on set \(S\), and \(\preceq_2\) is reflexive on set \(T\), \(s \preceq_1 s\) and \(t \preceq_2 t\).

So \((s, t) \preceq (s, t)\)

Therefore, \(\preceq\) is reflexive.

**Antisymmetric:** Let \(s, u \in S\) and \(t, v \in T\).

If \((s, t) \preceq (u, v)\) and \((u, v) \preceq (s, t)\)

then \(s \preceq_1 u\), \(t \preceq_2 v\), \(u \preceq_1 s\), and \(v \preceq_2 t\).

Since \(s \preceq_1 u\) and \(u \preceq_1 s\), and \(\preceq_1\) is antisymmetric

\(s = u\)

Since \(t \preceq_2 v\) and \(v \preceq_2 t\), and \(\preceq_2\) is antisymmetric

\(t = v\)

Therefore \((s, t) = (u, v)\)

So \(\preceq\) is antisymmetric.

**Transitive:** Let \(s, u, w \in S\) and \(t, v \in T\).

If \((s, t) \preceq (u, v)\) and \((u, v) \preceq (w, x)\)

then \(s \preceq_1 u\), \(t \preceq_2 v\), \(u \preceq_1 w\), and \(v \preceq_2 x\).

Since \(s \preceq_1 u\) and \(u \preceq_1 w\), and \(\preceq_1\) is transitive

\(s \preceq_1 w\)

Since \(t \preceq_2 v\) and \(v \preceq_2 x\), and \(\preceq_2\) is transitive

\(t \preceq_2 x\)
therefore, \( (s, t) \leq (w, x) \)

so \( \leq \) is transitive \( \boxed{\text{QED}} \)

4.1. 6. Show that if \( a, b, c \) and \( d \) are integers, where \( a \neq 0 \), such that \( a | c \) and \( b | d \), then \( ab | cd \)

Proof: Let \( a, b, c, d \in \mathbb{Z} \), where \( a \neq 0 \), such that \( a | c \) and \( b | d \).

\[ c = ka \quad \text{and} \quad d = jb \quad \text{for some integers} \; k, j \]

\[ cd = ka \cdot jb = (kj)ab \]

since \( kj \) is an integer

\( ab | cd \) \( \boxed{\text{QED}} \)