If \( f \) and \( f \circ g \) are 1-1, then \( g \) is also 1-1.

**Proof:** \( f \) and \( f \circ g \) are 1-1 functions.
\[
f(x) = f(y) \quad \text{and} \quad (f \circ g)(a) = (f \circ g)(b)
\]
\[
x = y \quad \text{and} \quad a = b
\]

**WTS:** \( g \) is 1-1

Let \( g(z) = g(w) \) differentiate
\[
f(g(z)) = f(g(w))
\]
\[
(f \circ g)(z) = (f \circ g)(w)
\]
\[
z = w
\]

Therefore \( g \) is 1-1.

R on set \( A \)

- **Reflexive** if \((a,a) \in R\) for all \( a \in A\)
- **Symmetric** if \((b,a) \in R\) when \((a,b) \in R\) for all \( a, b \in A\)
- **Anti-Symmetric** if \( a = b \) when \((a, b), (b, a) \in R\) for all \( a, b \in A\)
- **Transitive** if \((a, c) \in R\) when \((a, b), (b, c) \in R\) for all \( a, b, c \in A\)

**PART A:** \( a \) is taller than \( b \)

- Let \( R \) be a relation on set \( A \) defined by \((a, b) \in R\) if \( a \) is taller than \( b \) for any \( a, b \in A\).
  - \((a, a) \notin R\), so \( a \) cannot be taller than \( a \) or \( a \) is not reflexive.
  - \((a, b) \in R\) \( a \) is taller than \( b \)
  - \((b, a) \notin R\) \((b, a) \in R\) \( b \) is taller than \( a \)
  - \( a \) is not symmetric
  - \( b \) is not symmetric
  - \( R \) is anti-symmetric
  - \( R \) is not reflexive
\[ \text{let } a, b, c \in A, (a, b), (b, c) \in R \]
\[ a \text{ is taller than } b, b \text{ is taller than } c \]
\[ (a, c) \in R \]
\[ \text{so } R \text{ is TRANSITIVE} \]

\[ \text{PART B: } a \text{ & } b \text{ were born on the same day} \]
\[ \text{let } R \text{ be a relation on set } & \text{ all people } & \text{ defined by} \]
\[ (a, b) \in R \text{ if } & \text{ only if } a \text{ & } b \text{ were born on the same day} \]
\[ \text{for any } a \in A, a \text{ & } b \text{ are born on the same day} \]
\[ (a, a) \in R \text{ for every } a \in A \]
\[ \therefore R \text{ is reflexive} \]
\[ \text{let } a, b \in A, (a, b) \in R \]
\[ a \text{ & } b \text{ were born on the same day} \]
\[ b \text{ & } a \text{ were born on the same day} \]
\[ (b, a) \in R \]
\[ \therefore R \text{ is symmetric} \]
\[ \text{since different persons born on same day, } R \neq \text{ anti-symmetric} \]
\[ \text{let } a, b, c \in A, (a, b), (b, c) \in R \]
\[ a \text{ & } b \text{ were born on the same day} \]
\[ b \text{ & } c \text{ were born on the same day} \]
\[ a \text{ & } c \text{ were born on the same day} \]
\[ (a, c) \in R \]
\[ \therefore R \text{ is transitive} \]

\[ \text{So } R \text{ is reflexive, symmetric, \& transitive} \]

\[ \text{PART C: } a \text{ has the same first name as } b \]
\[ \text{let } R \text{ be a relation on set } & \text{ all people } & \text{ defined by} \]
\[ (a, b) \in R \text{ if } & \text{ only if } a \text{ & } b \text{ have same first name} \]
\[ \text{for any } a \in A, a \text{ & } b \text{ have same first name} \]
\[ (a, a) \in R \text{ for every } a \in A \]
\[ \therefore R \text{ is reflexive} \]
\[ \text{let } a, b \in A, (a, b) \in R \]
\[ a \text{ & } b \text{ have same first name} \]
\[ b \text{ & } a \text{ have same first name} \]
\[ (b, a) \in R \]
\[ \therefore R \text{ is symmetric} \]
\[ \text{different people may have same last name, } R \neq \text{ anti-symmetric} \]
Let \( a, b, c \in A, (a, b), (b, c) \in R \)  
\( a \neq b \) have same first name  
\( b \neq c \) have """"""""  
\( a \neq c \) have """"""""

\( (a, c) \in R \)  
\( \therefore R \) is transitive

So \( R \) is reflexive, symmetric, \( \theta \) transitive

PART D: \( a \neq b \) have a common grandparent

Let \( R \) be a relation on the set of all people \( A \) defined by \( (a, b) \in R \) if \( \& \) only if \( a \neq b \) have a common grandparent for any \( a \in A \).

\( (a, a) \in R \) for every \( a \in A \)  
\( \therefore R \) is reflexive

Let \( a, b \in A, (a, b) \in R \)  
\( a \neq b \) have a common grandparent  
\( b \neq a \) have a common grandparent  
\( (b, a) \in R \)  
\( \therefore R \) is symmetric

\( \) diff. people may have common grandparent so \( R \neq \) anti-symmetric

Let \( a, b, c \in A, (a, b), (b, c) \in R \)  
\( a \neq b \) have a common grandparent  
\( b \neq c \) have a common grandparent  
\( a \neq c \) may or may not have a common grandparent  
\( \therefore R \) is not transitive

So \( R \) is reflexive \& symmetric

\( b \)  
\( (x, y) \in R \) if \( \& \) only if \( R \neq TR \)

PART A: \( x + y = 0 \)  
\( (x, y) \in R \)

\( (1, 1) \notin R \)  
\( \therefore \) Not reflexive

Let \( x, y \in R, (x, y) \in R \)  
\( x + y = 0 \)  
\( y + x = 0 \)  
\( \therefore \) symmetric
. \((1, 1), (1, 1) \in \mathbb{R}\) but \(1 \neq 1\)  
  Not anti-symmetric

. \((2, -2), (-2, 2) \in \mathbb{R}\) but \((2, 2) \notin \mathbb{R}\)  
  Not transitive

So \(R\) is symmetric

PART B: \(x = \pm y\)

. \((x, y) \in \mathbb{R}\) if and only if \(x = \pm y\)
  \(x = x \quad \forall x \in \mathbb{R}\) \((x, x) \in \mathbb{R} \quad \forall x \in \mathbb{R}\)
  \(\therefore R\) is reflexive

. \(x, y \in \mathbb{R}, (x, y) \in \mathbb{R}\)
  \(x = \pm y\)
  \(y = \pm x\)
  \(\therefore R\) is symmetric

. \((1, 1), (1, 1) \in \mathbb{R}\) but \(1 \neq 1\)  
  Not anti-symmetric

. Let \(a, b, c \in \mathbb{R}, (a, b), (b, c) \in \mathbb{R}\)
  \(a = \pm b, b = \pm c\)
  \(a = \pm c\)
  \((a, c) \in \mathbb{R}\)  
  \(\therefore R\) is transitive

So \(R\) is Reflexive, symmetric & transitive

PART C: \(x - y\) is a rational number

. \((x, y) \in \mathbb{R}\) if and only if \(x - y\) is a rational number
  any \(x \in \mathbb{R}, x - x = 0\) is rational number
  \((x, x) \in \mathbb{R} \quad \forall x \in \mathbb{R}\)
  \(\therefore R\) is reflexive

. Let \(x, y \in \mathbb{R}, (x, y) \in \mathbb{R}\)
  \(x - y\) is rational number
  \(y - x\) is rational number
  \((y, x) \in \mathbb{R}\)
  \(\therefore R\) is symmetric

. \((2, 3), (3, 2) \in \mathbb{R}\) but \(2 \neq 3\)  
  Not anti-symmetric

. Let \(x, y, z \in \mathbb{R}, (x, y), (y, z) \in \mathbb{R}\)
  \(x - y, y - z\) are rational number
  \((x - y) + (y - z)\) are rational number
  \(x - z\) is rational number
  \((x, z) \in \mathbb{R}\)
  \(\therefore R\) is transitive.
22 \( \{ (a,a), (a,b), (b,c), (c,d), (a,a), (d,b) \} \equiv R \)

Elements \( \rightarrow \) Vertices \( \{ a, b \} \) where \( (a,b) \in R \rightarrow \) Edges

\( R \) is relation on \( \{ a, b, c, d \} \)

\( 5/5 \)

2b

\[ R = \{ (a,a), (a,b), (b,a), (b,b), (c,a), (c,c), (c,d), (d,d) \} \]

on \( \{ a, b, c, d \} \)

\( 5/5 \)