Math1090 Final Exam
Spring, 2011

Name ____________________________________________
Instructor:                                         28 minutes

Instructions:
  □ Show all work, as partial credit will be given where appropriate.
  □ If no work is shown, there may be no credit given.
  □ All final answers should be written in the space provided on the exam
    and in simplified form.
  □ Each numbered problem is worth 20 points.

DO NOT WRITE IN THIS TABLE!!!
(It is for grading purposes.)

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Raw Total (out of 200 points)

Total (percentage)
1. Write the equation of the line that passes through the point (5, -3) and is perpendicular to the line \(2x - 3y = 6\). (Put answer in slope-intercept form.)

\[-3y = 6 - 2x\]
\[y = -2 + \frac{2}{3}x\]
\[y + 3 = \frac{2}{3}(x - 5)\]
\[y = \frac{2}{3}x - \frac{10}{3} - 3\]

Line: \[y = \frac{2}{3}x - \frac{19}{3}\]
2. Margie makes a $500 contribution at the end of each quarter to a retirement account for 10 years earning 7% interest.
(a) How much money is in the account after the 10 years of contributions?

\[ r_c = \frac{0.07}{4} \]

Annuity

\[ S = 500 \left( \frac{(1 + r_c)^{10 \times 4} - 1}{r_c} \right) \]

Answer: $28617.07

(b) After the first 10 years, she makes no additional contributions and no withdrawals, and she leaves the money in the account for another 10 years. How much money is in the account at the end of those next 10 years?

Compound

\[ S = 28617.07 (1 + r_c)^{40} \]

Answer: $57279.85
3. Mr. Rogers makes and sells sweaters. His fixed costs are $1200 and it costs $8 to make each sweater. His revenue function is described by $R(x) = -x^2 + 88x$.

(a) How many sweaters should Mr. Rogers make to maximize profit?

$$C(x) = 1200 + 8x$$

$$P(x) = R(x) - C(x) = -x^2 + 80x - 1200$$

vertex $x = \frac{-b}{2a} = \frac{80}{-2}$

Number of sweaters: $x = 40$

(b) What is Mr. Rogers' maximum profit?

$$P(40) =$$

Maximum profit: $4400$

(c) How many sweaters must he sell to break even?

$$0 = -x^2 + 80x - 1200$$

$$x = \frac{-80 \pm \sqrt{6400 - 4(1200)}}{2} = 40$$

$X = 20, 60$

Number of sweaters to break even: $x = 20, 60$
4. Solve the following system of equations using Gaussian elimination.
\[
\begin{align*}
2x + 4y &= -4 \\
6x + 3z &= 12 \\
y + z &= -2
\end{align*}
\]

\[
\begin{bmatrix}
2 & 4 & 0 & -4 \\
6 & 0 & 3 & 12 \\
0 & 1 & 1 & -2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -2 \\
0 & 3 & 12 \\
0 & 1 & 1 & -2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -2 \\
0 & -12 & 3 & 24 \\
0 & 1 & 1 & -2
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -2 \\
0 & 1 & 1 & -2 \\
0 & 0 & 15 & 0
\end{bmatrix} \Rightarrow z = 0
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -2 \\
0 & 1 & 1 & -2 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 0 & -2 \\
0 & 1 & 0 & -2 \\
0 & 0 & 1 & 0
\end{bmatrix} \Rightarrow y = -2
\]

\[
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 0
\end{bmatrix} \Rightarrow x = 2
\]

Solution: \((2, -2, 0)\)
5. Solve these equations.

(a) \( \log_3(x+4) - \log_3(x+1) = \log_3 x \)

\[
\begin{align*}
\log_3 ((x+4)(x+1)) &= \log_3 x \\
(x+4)(x+1) &= x \\
x^2 + 5x + 4 - x &= 0 \\
x^2 + 4x + 4 &= 0 \\
(x+2)^2 &= 0
\end{align*}
\]
\(x = \text{No Solution}\)

(b) \(150e^{0.05t} - 40 = 410\)

\[
\begin{align*}
150e^{0.05t} &= 450 \\
e^{0.05t} &= 3 \\
\ln 3 &= 0.05t
\end{align*}
\]
\(t = \frac{20 \ln 3}{21.97}\)

(c) \(\left(\frac{2x+1}{x-2} + 4 = \frac{5}{x-2}\right)(x-2)\)

\[
\begin{align*}
2x + 1 + 4(x-2) &= 5 \\
4x - 8 &= 0 \\
6x - 7 &= 5 \\
6x &= 12 \\
x &= 2
\end{align*}
\]
\(x = \text{No Solution}\)
\(x > 2\)
6. You are in the business of manufacturing nordic skis. You make two kinds of skis, classic and skate. It costs you $100 to make a pair of classic skis, and $150 to make a pair of skate skis. Your daily production budget is $15,000. You need to make at least as many classic skis as skate skis, and you can't put out more than 100 pairs of classic skis in a day.

(a) Write down the inequalities that describe your constraints in making skis. Let \( x \) denote the number of pairs of classic skis and \( y \) the number of pairs of skate skis.

\[
\begin{align*}
(1) & \quad 100x + 150y \leq 15000 \\
(2) & \quad x \geq y \\
(3) & \quad x \leq 100 \\
(4) & \quad x \geq 0 \quad y \geq 0
\end{align*}
\]

Classics = \( x \)  
Skate = \( y \)

(b) Graph these inequalities, label the axes, shade in the feasibility region, and label the vertices.

(c) Suppose that you can model your daily profit by the function \( P(x,y) = 20x + 25y \). How many pairs of each type of ski should you try to manufacture in a day in order to maximize profit?

\[
\begin{align*}
(0,0) = 0 & \quad (100,33.33) = 2833.33 \\
(60,60) = 2700 & \quad (100,0) = 2000
\end{align*}
\]

Number of classic ski pairs: 100  
Number of skate ski pairs: 33 (rounded)
7. (a) Describe the transformations (stretches, shrinks and/or reflections) for this function \( h(x) = -2(x - 1)^2 + 3 \). Compare it to the base graph of \( f(x) = x^2 \).

- Shift(s) (if any): **Right by 1, up by 3**
- Stretch/shrink (if any): **Vertical Stretch by 2**
- Reflection(s) (if any): **Vertical reflection**

(b) Below is the graph of \( f(x) = x^2 \). On these same axes, graph the function \( h(x) \) given in part (a).
8. Let $A = \begin{bmatrix} 2 & 3 \\ 9 & 7 \\ -3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 12 \\ 2 & -5 \\ 8 & 11 \end{bmatrix}$, $C = \begin{bmatrix} 8 & -7 \\ -4 & 2 \end{bmatrix}$. Compute the following, or explain why it is not possible.

(a) $A + 2B = \begin{bmatrix} -4 & 27 \\ 13 & -3 \\ 13 & 28 \end{bmatrix}$

(b) $C - 4B = \begin{bmatrix} -72 & 48 \\ 36 & -24 \\ 20 & -34 \end{bmatrix}$

(c) $BC = \begin{bmatrix} -2y - 48 & 21 + 2y \\ 16 + 20 & -14 - 10 \\ 6y - 8y & -56 + 22 \end{bmatrix}$
9. Let \( f(x) = x^2 + 1 \), \( g(x) = \frac{x}{x+3} \), \( h(x) = 6x + 7 \). Find the following.

(a) Domain of \( g(x) \): \( x \neq -3 \)

(b) \( (fh)(2) = \frac{95}{9} \) \( f(2) = 5 \) \( h(2) = 19 \)

(c) \( (f \circ h)(x) = \frac{36x^2 + 84x + 50}{10x - 1} \)

(d) \( h(x) + f(x) + 3 = x^2 + 6x + 1 \)

(e) \( g^{-1}(x) = \frac{-3x}{x-1} \)

\[ x = \frac{y}{y+3} \]

\[ xy + 3x - y = 0 \]

\[ y(x - 1) = -3x \]

\[ y = \frac{-3x}{10x - 1} \]
10. Jordan just bought a home. He borrowed $210,000 at a fixed rate of 5.1% interest, compounded monthly, for 25 years.

(a) What monthly payment does he make?

\[ r_c = \frac{0.051}{12} \]

\[ R = \frac{r_c}{(1 - (1 + r_c)^{-12 \times 25})} \]

\[ R = \frac{1239.91}{210,000} \]

Monthly payment: \[1239.91\]

(b) If he decides to sell the house after making exactly 60 payments, what is his payoff for the loan?

\[ N = 300 \]

\[ S_{300-60} = 1239.91 \left( \frac{1 - (1 + r_c)^{-240}}{r_c} \right) \]

Loan payoff: \[186314.66\]