Addition, Multiplication and Scalar Multiplication of Matrices

Math 1090-001 (Spring 2001)  Friday, Feb. 9, 2001

Exercises

1. Give an example of two matrices $A$ and $B$ for which $AB$ is defined but not $BA$.

2. Suppose $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{p \times q}$. We know that $AB$ is defined if and only if $n = p$. Suppose also this is the case, what are the dimensions of $AB$?

3. Find $B$ if $2A - 3B + C = 0$, where

$$A = \begin{bmatrix} -1 & 3 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}.$$ 

4. Show that $AB \neq BA$, where

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}.$$
Problems (not for submission)

1. Suppose \( A \in \mathbb{R}^{m \times n} \) is such that \( A + B = B \) for every \( B \in \mathbb{R}^{m \times n} \). Find \( A \).

2. A matrix \( A \in \mathbb{R}^{m \times n} \) is said to be a zero matrix if all of its entries are zero. If \( B \in \mathbb{R}^{m \times n} \), what is \( A + B \) ? \( B + A \)?

3. Recall that if \( x, y \in \mathbb{R} \) are such that \( xy = 0 \), then either \( x = 0 \) or \( y = 0 \). However, matrix multiplication does not have this property. Give an example of two nonzero matrices \( A \) and \( B \) such that \( AB \) is a zero matrix.

4. Textbook problems: 3.2.43 to 3.2.46.

5. Read about the identity matrix on p.233 of the textbook.

6. Show what \( AB \neq BA \), where

\[
A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}.
\]

7. Solve for \( x, y, z \) and \( w \):

\[
\begin{bmatrix} x & 4 \\ 4y & w \end{bmatrix} - \begin{bmatrix} 4x & 2z \\ -3 & -2w \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ y & 6 \end{bmatrix}.
\]

8. Rewrite the matrix equation the following matrix equation as a linear system of linear equations:

\[
\begin{bmatrix} 3 & 1 & 0 \\ 2 & -2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}.
\]

Solve the system in matrix notation.

9. Textbook problems: 3.3.3 to 3.3.10, 3.3.22, 3.3.26, 3.3.38, and 3.3.45.