Let us practice!
Mathematics 1210-4 (Calculus I)

This is just a practice exam. For the real exam, you will have at the very most 3 problems.

Problem n° 1. Evaluate each of the following limits or state that they do not exist.

i) \( \lim_{x \to 2^+} (x - [x]) \); \( \lim_{x \to 2^-} ([x] - x) \); \( \lim_{x \to -\infty} \left( \frac{\cos x}{x + \pi} \right) \)

ii) \( \lim_{x \to 2} \frac{x - 2}{\sin(\pi x)} \); \( \lim_{x \to 3^-} \frac{\cos(x - 3)}{x - 3} \); \( \lim_{x \to \infty} \left( \frac{2x + 1}{\sqrt{3x^2 + 1}} \right) \)

iii) \( \lim_{x \to -\infty} \left( \sqrt{2x^2 + 3x} - \sqrt{2x^2 - 5} \right) \); \( \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^{1000} \)

iv) \( \lim_{x \to -\infty} \sqrt{\frac{2x^2 - 3x}{5x^2 + 1}} \); \( \lim_{\tau \to (\pi/2)^+} \frac{\pi \tau}{\cos \tau} \); \( \lim_{x \to 1} \left( \frac{1 - \cos(2\pi x)}{\pi^2(x - 1)^2} \right) \); \( \lim_{x \to 0} \frac{x^2 \sin(\frac{1}{x})}{\sin x} \).

Problem n° 2. Find the oblique asymptotes (if they do exist) for

i) \( f(x) = \frac{3x^3 + 4x^2 - x + 1}{x^2 + 1} \),

ii) \( f(x) = \frac{3x^3 + x^2 - 3x + 2}{3x^2 + 1} \).

Problem n° 3. Find the asymptotes to the graph of

i) \( f(x) = \frac{2x - 1}{x^2 - 1} \),
ii) \( f(x) = \frac{3}{4x^2 - 1} \),

iii) \( f(x) = \frac{3x^2}{4x^2 - 1} \).

**Problem n° 4.** Give the points of discontinuity of each of the following functions.

i) \( f(x) = \frac{x}{(x-2)(x-4)} \),

ii) \( f(x) = x^2 \sin \left( \frac{1}{x} \right) \), if \( x \neq 0 \) and \( f(0) = 0 \),

iii) \( f(x) = x^2 \sin \left( \frac{1}{x} \right) \), if \( x \neq 0 \) and \( f(0) = 1 \),

iv) \( f(x) = \sqrt{(x-3)(6-x)} \), \( 3 \leq x \leq 6 \).

**Problem n° 5.** Prove (using the Intermediate Value Theorem) that the equation

\[ x^3 + 3x - 2 = 0 \]

has a real solution between 0 and 1.

**Problem n° 6.** Let

\[
 f(x) = \begin{cases} 
 -1 & \text{if } x \leq 0, \\
 ax + b & \text{if } 0 < x < 1, \\
 1 & \text{if } x \geq 1. 
\end{cases}
\]

Determine \( a \) and \( b \) so that \( f \) is continuous everywhere.

**Problem n° 7.** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4 degrees and 6.5 degrees. How far apart are the ships?